Image Segmentation based on Fuzzy Region Competition

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***Abstract*—In this work, we studied variational models for image segmentation present in literature, in which will be discussed its mathematical formulations and characteristics of minimization processes. Among these techniques, it will be explored the method Fuzzy Region Competition and segmentation models that can be derived from its energy functional. Inspired by this method, three modifications were proposed based on these models that minimize some of their limitations and which emerges as alternatives for real-world image segmentation. The proposed modifications were validated by the good results obtained using natural, textured and noisy images. In addition, some comparative results are shown with other similar techniques with the objective of reporting the effectiveness of the proposed models.** [1]

***Keywords*-soft image segmentation, Fuzzy Region Competition, Variational Methods, Partial Differential Equations.**

I. Introduction

Image segmentation is a fundamental field in image processing and computer vision. Despite intense studies in last decades, image segmentation problem still presents an important research task. In literature, there exists a large diversity of segmentation techniques and researchers have made efforts to develop efficient and consistent algorithms. Methods that incorporate variational principles appear with powerful tools to solve several problems related to image segmentation. In a variational approach, the segmentation is obtained by minimizing an energy functional formulated under a homogeneity criterion of the image regions.

In this sense, we may distinguish between hard (also called crisp) and soft segmentation approaches. The goal of hard segmentation is to subdivide the image domain into its non-overlapping and connected components. The main examples are *Mumford-Shah* functional [1] and *Snake* model [2], which boosted the development of region-based and contour-based variational segmentation models, respectively. The first one deals with image smoothing and boundary preservation simultaneously and the solution is a piecewise smooth approximation from the reference image. The latter uses a parametric dynamic curve, explicitly represented, that moves from an initial position towards to the image object boundaries. The most of further developed image segmentation models that uses a variational formulation are based on *Mumford-Shah* and *Snake* method [3] [4]. However, generally these methods compute local solutions and present a slow convergence process, since they are based on curve evolution techniques that are numerically solved through Euler-Lagrange Equations associated with gradient-descendent schemes.

Mory and *Ardon* [5] proposed an extension of the classical *Region Competition* algorithm [6] to a $soft$ ($fuzzy$) approach, named *Fuzzy Region Competition* (FRC), which aims to segment an image in two regions (background and foreground). This method describes the image regions by means of a statistical criterion, which is used for the competition procedure between these image regions, coming from Bayesian principles [6], constants regions [3] or regions with intensity variations [1]. Furthermore, the refered method can assume a supervised or an unsupervised approach. Its energy functional is convex related to the fuzzy membership function, in which global solutions that minimizes its functional and stable segmentation models can be obtained.

However, some of these models presented in *Mory* and *Ardon* work's [5] have drawbacks related to its practical applications and to the statistical criteria used to characterize the image segmentation models. These limitations motivated us to propose three alternative image segmentation models based on FRC methodology.

Constant Region Competition (CRC) [5] model represents each image region by a constant value, which is not indicated for segmenting textured images. The first proposed modification is a local extension of CRC that computes these constants by considering the influence of neighborhood in relation to each image point. *Mory* and *Ardon* proposed in [5] a supervised segmentation model that uses a probability distribution to represent image regions. This model has practical limitations due to the necessity of the sampling of image regions before the segmentation process. In this sense, we developed an unsupervised version that turns the segmentation process automatic, keeping the quality of the results. The third proposed method refers to an extension of the two-phase FRC to multiphase image segmentation. The main reason to develop this extension is related to the applicability of *Mory* and *Ardon* method, which is restricted to images composed by two regions, knowing that we can naturally find images

[1]Master thesis of Vinicius Ruela Pereira Borges

constituted of several regions. Moreover, multiphase image segmentation models based on FRC methodology are not very explored in the scientific community.

This paper is organized as follows: Section II presents the two-phase *Fuzzy Region Competition* energy functional and the segmentation models that can be derived from its methodology. Section III describes the *Local Weighted Constant Region Competition* method, which is the local extension of the CRC model proposed by *Mory* and *Ardon*. Section IV presents the second proposed modification, an unsupervised case of *Fuzzy Region Competition* which uses probability density functions to describe images. Section V shows the multiphase image segmentation model based on *Fuzzy Region Competition*. Experimental results are reported after each segmentation model description in its respective sections. Section VI shows the conclusions about the proposed image segmentation modifications.

II. Two-Phase Fuzzy Region Competition

With the success of the *Region Competition* algorithm [6], several variational region-based methods that use its ideas have been developed. *Mory* and *Ardon* proposed the two-phase *Fuzzy Region Competition* (FRC) framework [5], which has a $fuzzy$ approach and aims to segment an image I over the domain $\Omega \subset \mathbb{R}^m$ in two regions based on its intensity distributions. The FRC minimization problem, in a general formulation, is given by:

$$\min_{u \in BV(\Omega)_{[0,1]}, \boldsymbol{\alpha}} \left\{ F_{FRC}(u, \boldsymbol{\alpha}) = \int_\Omega g|\nabla u| + \right.$$
$$\left. + \int_\Omega u(x) r_1(\alpha_1, x) dx + \int_\Omega (1 - u(x)) r_2(\alpha_2, x) dx \right\} \quad , \quad (1)$$

where $u \in BV(\Omega)_{[0,1]}$ (set of bounded variation functions between $[0, 1]$) is a variable that represents the fuzzy membership function. g is a positive boundary function, decreasing with the image gradient. $r_i : \Omega \to \mathbb{R}$, for $i = 1$ or 2, are error functions, which design image regions using intensity properties. They are dependent on a set of region parameters $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)$, assuming to be scalars [6], constants [3] or space varying functions [1]. When $\boldsymbol{\alpha}$ is known *a priori*, the problem in Eq. (1) becomes a supervised segmentation model, otherwise the segmentation model is said to be unsupervised, where $\boldsymbol{\alpha}$ needs to be optimized. The first term in energy functional of Eq. (1) is responsable for smoothing the variable u, while the remainder terms are related to the cost of assign each domain point to the respective regions.

The error functions $r_i(\alpha_i, x)$ characterize image regions using the statistical parameters α_i. We can obtain several segmentation models using different error functions, where are commonly employed:

- $r_i(\alpha_i, x) = -\lambda \log(P_i(I(x)|\alpha_i))$, where $P_i(I(x)|\alpha_i)$ is a probability distribution with scalar parameters α_i ($\alpha_i \in (\mathbb{R}^k)^2$) [6]. *Mory* and *Ardon* proposed a supervised image segmentation model by the use of this error function on the functional of Eq. (1);
- $r_i(\alpha_i, x) = \lambda(I(x) - c_i)^2$, where parameters $\alpha_i = c_i$ are constant values ($c_i \in \mathbb{R}^2$) [3]. *Mory* and *Ardon* used this error function in the functional of Eq. (1) to obtain the *Constant Region Competition* (CRC) method;
- $r_i(\alpha_i, x) = \lambda(I(x) - s_i(x))^2 + \zeta|\nabla s_i|^2$, where parameters $\alpha_i = s_i$ are space varying functions ($s_i \in (\mathcal{C}^1)^2$) [1]. The derived segmentation model is refered as *Smooth Region Competition* (SRC).

remarking that λ is a regularizer parameter for balancing the smoothness and the competition.

The functional in Eq. (1) is convex with respect to u, so minimizing it, optimal global solutions regardless of initial conditions can be obtained in the supervised case. In the unsupervised case, the obtained solutions are weakly sensitive to initial conditions, since it involves the optimization of region parameters. Generally, the minimization of the problem in Eq. (1) consists of performing alternately the two steps below:

(i.) Keeping u fixed, compute optimal values for the parameters $\boldsymbol{\alpha}$, according to the adopted error function. This step takes place only in the unsupervised segmentation case;

(ii.) Keeping $\boldsymbol{\alpha}$ fixed, update fuzzy membership function u by minimizing Eq. (1);

where function u is initialized randomly with the restriction $u(x) \in [0, 1]$, for each $x \in \Omega$. This minimization process ends when no significant changes occurs in u or if we fix a limited number of iterations.

It is important to emphasize that step (ii.) can be numerically solved using the Euler-Lagrange equations associated with a gradient-descendent scheme. However, this strategy does not take advantage over the convexity of functional of Eq. (1). An alternative strategy is related to the use of *Chambolle*'s $dual$ projection algorithm [7], which is a fast and stable numerical tool to minimize energy functionals that incorporate total variation regularizing terms. To achieve this objective, we use *Bresson's* strategy [8], in which the segmentation problem in Eq. (1) can be rewritten by the approximation below:

$$\min_{(u,v) \in BV(\Omega)_{[0,1]}} \left\{ F_{AP}(u, v) = \int_\Omega |\nabla u| \right.$$
$$\left. + \frac{1}{2\theta} \int_\Omega |u - v|^2 + \int_\Omega rv + \rho\nu(v) \right\} \qquad (2)$$

where v is an auxiliary function, $r = r_1 - r_2$ is the competition function, $\nu(\varepsilon) = max(0, |2\varepsilon - 1| - 1)$, $\rho > \frac{1}{2}|r|_\infty$ and θ is chosen to be a small enough value, so that the minimizing pair (u^*, v^*) are almost identical in relation to L^2 norm. The functional in Eq. (2) depends only on the function u in the two first terms, then *Chambolle's* $dual$ projection algorithm can be used for F_{AP} minimization. Based on this fact, the solution of the functional in Eq. (2) is reached by carrying out sucessive minimization steps in u and v, alternately as:

1) Keeping u fixed, compute v:

$$v = max\{min\{u - \theta r, 1\}, 0\} \tag{3}$$

2) Keeping v fixed, compute u:

$$u = v - \theta div\mathbf{p} \tag{4}$$

where the vector $\boldsymbol{p} = (p^1, p^2) \in \mathcal{C}_c^1(\Omega, \mathbb{R}^m)$ can be calculated by the fixed point algorithm, iterating for $n \geq 0$:

$$\mathbf{p}^{n+1} = \frac{\mathbf{p}^n + \tau\nabla(div\mathbf{p}^n - v/\theta)}{1 + \tau|\nabla(div\mathbf{p}^n - v/\theta)|} \tag{5}$$

where $\boldsymbol{p}^0 = \mathbf{0}$ and $\tau < 1/8$ guarantees the numerical stability of the scheme and g is obtained as

$$g(|\nabla I|) = \frac{1}{1 + \beta|\nabla I|^2} \tag{6}$$

where β is a positive constant that controls the gradient influence in function g and I is the reference image.

The *Chambolle's dual* projection algorithm has numerical stability in the minimization process, fast minimization and it is advantageous for the convergence process.

In the next sections, the three proposed modifications for image segmentation based on FRC method will be detailed. These sections contains the idea behind the derived models, motivation and a segmentation result that illustrates the usage of the respective model.

III. Locally Weighted Constant Region Competition

Constant Region Competition (CRC) model is very useful for segmenting images in which its regions are homogeneous or corrupted by noise. On the other hand, the applicability of this model in real and natural images is too restricted, since images with homogeneous regions are rarely found in practice. Because this method computes optimal constants for each region, information like object texture and topology are disregarded during the segmentation process, since these parameters are computed from image global analysis. Using a local methodology over the image and performing the competition between the two regions considering only neighborhood points, the final segmentation result should be more effective.

Using this idea, we propose a segmentation model based on FRC that aims to approximate each image region by a constant value using local weighted analysis. The segmentation model is called *Locally Weighted Constant Region Competition* (LWCRC) [9], which uses the following error function:

$$r_i^{c_i}(x) = \lambda \int_{y\in\Omega} \omega(x - y)(I(x) - c_i)^2 dy. \tag{7}$$

where $\omega : \Omega \to \mathbb{R}^+$ is a window function, which provides the neighborhood influence in relation to each domain point and $\omega(x) \to 0$ when $|x| \to +\infty$. Tipically, ω can be defined as a normalized isotropic Gaussian window:

$$\omega(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-|x|^2}{2\sigma^2}\right), \tag{8}$$

where σ is the standard deviation, responsable for providing the scale notion in the image analysis. Further details about how this error function was determined is given in [9]. Replacing the error in Eq. (7) into the functional of Eq. (1) becomes:

$$\min_{u\in BV(\Omega)_{[0,1]}, (c_1,c_2)\in\mathbb{R}} \Bigg\{ F_{LWCRC}(u, c_1, c_2) = \int_\Omega |\nabla u| + \\ +\lambda \int_{x\in\Omega} u(x) \int_{y\in\Omega} \omega(x-y)(I(x) - c_1)^2 dy dx + \\ +\lambda \int_{x\in\Omega} (1 - u(x)) \int_{y\in\Omega} \omega(x-y)(I(x) - c_2)^2 dy dx \Bigg\}, \tag{9}$$

where $\boldsymbol{\alpha} = \{c_1, c_2\}$ and λ balances the error region terms and the total variation regularization. Keeping u fixed, the optimal values for c_1 and c_2 are obtained computing Euler-Lagrange Equations for equation (9). So, c_1 and c_2 are given by:

$$\begin{aligned} c_1^* &= \frac{\int_{y\in\Omega}\int_{x\in\Omega}(I(x)u(x))\omega(x-y)dxdy}{\int_{y\in\Omega}\int_{x\in\Omega}u(x)\omega(x-y)dxdy} \\ c_2^* &= \frac{\int_{y\in\Omega}\int_{x\in\Omega}(I(x)(1-u(x)))\omega(x-y)dxdy}{\int_{y\in\Omega}\int_{x\in\Omega}(1-u(x))\omega(x-y)dxdy}. \end{aligned} \tag{10}$$

Eqs. (10) can be rewritten by means of normalized convolutions [9], where such convolutions, in fact, create smooth approximations for each image region, considering the fuzzy membership functions u and $(1 - u)$ as certainty measures. Therefore, the constants c_1 and c_2 in our model are locally weighted averages, which differs from the global constants of the CRC model.

The minimization process of Eq. (9) takes place as in FRC unsupervised models, following basically the steps below: (i) Keeping u fixed, compute the optimal values for c_1^* and c_2^* by Eqs. (10); (ii) Keeping c_1^* and c_2^* fixed, update fuzzy membership function u minimizing the functional of Eq. (9) with respect to u using *Chambolle's dual* projection algorithm. The final segmentation is $c_1u + c_2(1 - u)$.

In Figure 1, we give an example of the proposed segmentation model. We aim to segment the central circle, shown in Figure 1(a), from the background. Figure 1(b) shows the initial state of the fuzzy membership function u. Figures 1(c) and 1(d) illustrate the intermediary state and the final state of function u after 150 time steps CRC model, respectively. Figures 1(e) and 1(f) show the intermediary state and the final state after 150 time steps of function u by LWCRC model, respectively. The parameters used in this experiments were $\theta = 0.15$, $\tau = 0.1$, $\lambda = 0.01$ and $\sigma = 0.04$. It can be emphasized that our model was able to identify the two textured objects correctly.

IV. Unsupervised Fuzzy Region Competition using Global Distribution Probabilities

In [5], *Mory* and *Ardon* explored a supervised approach of the FRC method aiming to analyse the functional convexity.

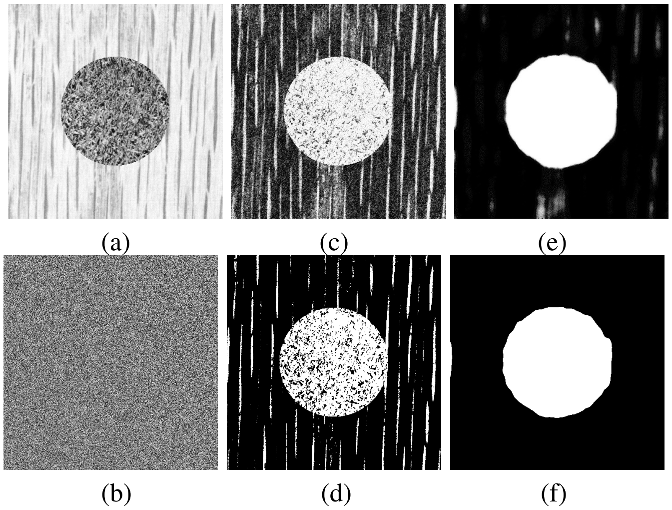


Figure 1: Image (256×256) constructed using Brodatz textures [10]: (a) Original image; (b) Initial u; (c-d) Intermediary and final state of u by CRC model, respectively; (e-f) Intermediary and final state of u by LWCRC model, respectively.

In this supervised model, we first should sample each image region, compute the respective parameters using the samples and estimate the probability distributions for these regions. The probability distributions of the two regions are used in the competition process, where a log-likelihood test determines the most similar region for each domain point.

Although good results that can be reached from the supervised case of FRC method, the segmentation process is not totally automatic. The necessity of a previous knowledge of the image regions limits the applicability of the segmentation model to problems in which this information is really available. Furthermore, the parameters estimation using the region samples might not significantly represent the entire region, which may lead to poor results. The remainders unsupervised FRC models, CRC and SRC, describe each image region by either a constant value, or a smooth approximation, which are inappropriate for textured images.

Trying to minimize these drawbacks, we proposed an unsupervised segmentation model based on FRC method that uses probability density functions (pdf's) to design image regions and to set a homogeneity criterion for the competition between regions [11]. Assuming that the probability distributions functions $P_i(I|\alpha_i)$ are known, replacing the error function $r_i(\alpha_i, x) = -\lambda \log(P_i(I(x)|\alpha_i))$ into the functional of the Eq. (1) leads to:

$$\min_{u \in BV(\Omega)_{[0,1]}, \boldsymbol{\alpha}} \Big\{ F_{UFRC}(u, \boldsymbol{\alpha}) = \int_\Omega g|\nabla u| \\ -\lambda \int_\Omega u(x) \log(P_1(I(x)|\alpha_1))dx \\ -\lambda \int_\Omega (1-u(x)) \log(P_2(I(x)|\alpha_2))dx \Big\}. \quad (11)$$

A typical model that can be used in the probability distribution function P_i is the Gaussian distribution, given by:

$$P_i(I(x)|\{\mu_i, \sigma_i\}) = \frac{1}{\sigma_i\sqrt{2\pi}} \exp\left(-\frac{(I(x)-\mu_i)^2}{2\sigma_i^2} \right) \quad , (12)$$

where P_i describes statistically the image regions using parameters $\alpha_i = \{\mu_i, \sigma_i\}$, where μ_i is the mean and σ_i is the standard variation of region i. So, the optimal values for the parameters of region 1 (α_1) and region 2 (α_2) can be obtained computing the Euler-Lagrange equations of the functional in Eq. (11) related to these parameters:

$$\mu_1^* = \frac{\int I(x)u(x)dx}{\int u(x)dx} \qquad \mu_2^* = \frac{\int I(x)(1-u(x))dx}{\int (1-u(x))dx} \quad (13)$$

$$\sigma_1^* = \sqrt{\frac{\int u(x)(I(x)-\mu_1)^2 dx}{\int u(x)dx}} \\ \sigma_2^* = \sqrt{\frac{\int (1-u(x))(I(x)-\mu_2)^2 dx}{\int (1-u(x))dx}} \quad (14)$$

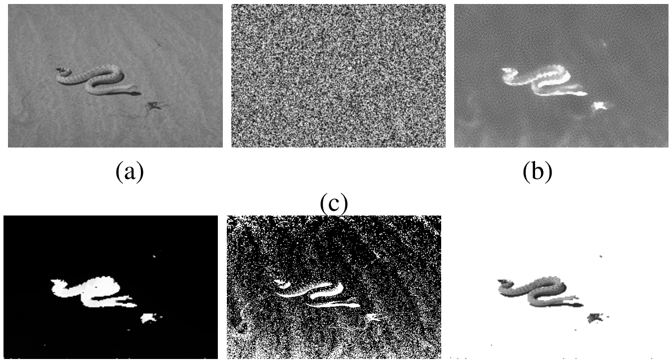
in which the fuzzy membership functions u and $(1-u)$ are certainty measures related to each image region.

The minimization problem of Eq. (11) can be solved following the generic scheme presented in Section II: (i) Keeping u fixed, compute the optimal values for μ_1 and μ_2 using Eq. (13) and, σ_1 and σ_2 by Eq. (14) and estimate the probability distributions P_1 and P_2 by Eq. (12); (ii) Keeping $\{\mu_1,\sigma_1\}$ and $\{\mu_2,\sigma_2\}$ fixed, update fuzzy membership function u by minimizing the functional in Eq. (11).

Figure 2 is a difficult segmentation task due to the high similarity between foreground and background intensities. We want to separate the snake from the background. Figure 2(b) shows the initial function u. Function u in intermediary state (step 20) is presented in Figure 2(c) and Figure 2(d) shows the final u after 700 time step iterations. Figure 2(e) illustrates function u obtained by CRC method. The reconstructed image $I_{REC} = Iu$, obtained from final function u (Figure 2(d)) is presented in Figure 2(f). It can be seen that snake was correctly segmented, while CRC model presented an erroneous result. In this experiment, the parameters were fixed as $\tau = 0.1$, $\theta = 0.15$, $\lambda = 0.4$ and $\beta = 0.005$.

V. Selective Multiphase Fuzzy Region Competition

The most of developed variational multiphase segmentation models [13] [14] [15] compute solutions that are sensitive with respect to initial conditions. Using this fact as motivation, we propose a multiphase image segmentation model which is weakly sensitive to initial conditions and is based on several runs of the *two-phase FRC* technique. The segmentation process takes place under a soft approach and its final result are hard partitions. So, we can avoid the occurrence of overlapping regions (points belonging to more than one region)



(a) (c) (b)

(d) (f) (e)

Figure 2: Snake image (321×481) from *Berkeley Segmentation dataset* [12]: (a) Original image; (b) Initial function u; (c) Intermediate state of function u; (d) Final function u after 700 steps; (e) Final function u obtained by CRC model; (f) Reconstructed image $I_{REC} = Iu$.

and vacuum (pointS without region label) in the image domain.

In this section, we present the *Selective Multiphase Fuzzy Region Competition (SMFRC)* model that uses a supervised approach and a statistical criterium to describe image regions. Specifically, we represent each image region by probability distributions, which are computed based on the statistical parameters extracted from the samples of the image regions. In the supervised case, the region parameters are computed from these samples and are known before the segmentation process.

The goal of the proposed model is to segment an image domain Ω in N regions, each one represented by a hard partition, in a way that

$$\Omega = \cup_{i=0}^{N} \Omega_i \text{ and } \Omega_i \cap \Omega_j = \emptyset. \tag{15}$$

where $\Omega_0 = \emptyset$.

The key idea behind the proposed segmentation model is to perform the two-phase FRC model $N-1$ times and to compute one fuzzy membership function per round. In each round i, the determined fuzzy membership function u_i, computed into $\Omega \backslash \cup_{j=0}^{i-1} \Omega_j$, is transformed into a hard partition using a threshold procedure. The N^{th} hard partition is taken as the complement of the set formed by the union of the $N-1$ partitions related to Ω. The proposed technique is soft during the segmentation process, but presents hard partitions as the final results.

In other words, we first determine u_1 and $(1-u_1)$ functions using the two-phase FRC model, where u_1 is the membership of the first region and $(1-u_1)$ is the membership for the background of the first region. This "first background" is composed by $N-1$ regions with different pdf's, however, only one pdf will be used to represent the image background. After this, we perform a threshold in u_1 to obtain Ω_1 and seek for u_2 and $(1-u_2)$ functions, only on the domain points that do not belong to partition Ω_1 (i. e., $\Omega \backslash \Omega_1$). Applying the same procedure described above and defining the "second background", function u_3 will be computed on domain points that do not belong to partition Ω_1 or Ω_2 (i. e., $\Omega \backslash (\Omega_1 \cup \Omega_2)$). This process continues until partition Ω_{N-1} has been computed. The N^{th} hard partition is obtained as

$$\Omega_N = \Omega \backslash \cup_{i=0}^{N-1} \Omega_i. \tag{16}$$

It can be seen that constraints in Eq. (15) are satisfied, once all domain points will have an unique region label after the determination of the N partitions.

The segmentation process above can be formulated as a minimization of the $N-1$ functionals, where each minimization procedure has the objective of determining the fuzzy membership function u_i, for $i = 1, ..., N-1$. The proposed model consists of minimizing the following energy functional:

$$\begin{aligned} F_i(u_i, \boldsymbol{\alpha}) &= \int_{\Omega'_i} g|\nabla u_i| + \int_{\Omega'_i} u_i(x) r_i(\alpha_i, x) dx \\ &+ \int_{\Omega'_i} (1 - u_i(x)) r_{s_i}(x) dx \quad , \end{aligned} \tag{17}$$

where $r_i(\alpha_i, x)$ is an error function defined as in the two-phase *Fuzzy Region Competition* framework. $\boldsymbol{\alpha}$ is a fixed region parameters set, defined as $\{\alpha_1, ..., \alpha_N\}$. Ω'_i is an image subdomain, used to fix the domain points that will participate of the computation of function u_i. For $i = 1, ..., N-1$, this subdomain is obtained as

$$\Omega'_i = \Omega \backslash \cup_{j=0}^{i-1} \Omega_j \tag{18}$$

knowing that $\Omega_0 = \emptyset$ and

$$\Omega_i = \{x \in \Omega'_i | u_i(x) > T\} \tag{19}$$

defines the hard partition containing the domain points which belongs to region i. $T \in [0, 1]$ is a threshold value.

As before mentioned, the proposed model approximates image regions by probability distributions. So, using the error function $r_i(\alpha_i, x) = -\lambda \log(P_i(I(x)|\alpha_i))$, we design r_{s_i} in a way that region i always competes with a region j which one has the highest probability of all other remaining regions. Thus, r_{s_i} is given by

$$r_{s_i}(x) = -\lambda \log(P_s(I(x)|\alpha_s))) \tag{20}$$

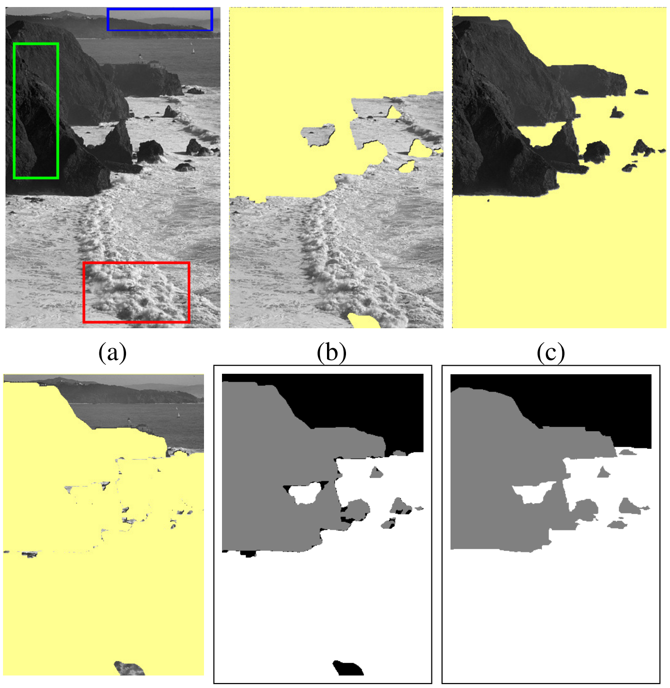
where

$$s = \arg\, max_{j=i+1,...,N}(P_j(I(x)|\alpha_j)) \quad , \tag{21}$$

where the probability distributions are Gaussians.

The energy functional in Eq. (17) is minimized with respect to u_i by the use of *Chambolle's dual* projection algorithm, as in the two-phase *Fuzzy Region Competition* model. More details about the algorithm of the proposed multiphase model can be encountered in [16].

In Figure 3, a result obtained by segmenting a nature image from *Berkeley Segmentation Dataset* [12] in three regions is showed. Figure 3(a) shows the original image with the red, green and blue rectangles representing each region manually sampled. Figures 3(b-c-d) present the reconstructed image regions using the hard partitions obtained by the proposed model. Figures 3(e) and 3(f) illustrate a comparison between the obtained segmentation and an adapted ground-truth to three-phase segmentation from *Berkeley Segmentation Dataset*. In this experiment, we set the parameters as: $\theta = 0.05$, $\tau = 0.1$, $\lambda = 0.2$, $\beta = 0.0001$, $N = 20$, $T = 0.5$ and 350 iterations for obtaining each partition. It is worth noting that the obtained segmentation is near from the adapted ground-truth provided by the Berkeley dataset developers.



(a) (b) (c)

(d) (e) (f)

Figure 3: Image (481×321) from [12] for three-phase segmentation: (a) Original image and region samples; (b) Reconstructed image using hard partition Ω_1; (c) Reconstructed image using hard partition Ω_2; (d) Reconstructed image using hard partition Ω_3; (e) Overall segmentation obtained by the proposed model; (f) Adapted Ground-truth from [12].

VI. Conclusion

In this paper, we described three variational image segmentations techniques which is based on *Fuzzy Region Competition* (FRC) technique. The first proposed modification is a local extension of the constant case of FRC framework (*Constant Region Competition* - CRC). This model represents each image region by a single constant value and uses a window function to weigth locally the competition between two regions. The proposed modification improved the segmentation quality compared with CRC using textured images.

The second modification is an unsupervised two-phase image segmentation model that describes image regions by probability distributions and guides the competition procedure by log-likelihood tests. The proposed model is more suitable for dealing with textured and natural images than CRC and can keep the segmentation quality when compared with supervised FRC method.

The third proposed model is a supervised multiphase segmentation algorithm which consists of performing the *two-phase* FRC $N-1$ times, where in each time, the obtained fuzzy membership function is transformed into a hard partition. Results showed that this model is promising considering that multiphase segmentation based on FRC is recent and needs to be more explored.

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