

Nullable, First and Follow

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$S \rightarrow \text{if } E \text{ then } S \ X$

$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

Esta é a gramática livre de contexto!

S $\rightarrow$ **if** E **then** S X

S $\rightarrow$ cmd

X $\rightarrow$

X $\rightarrow$ **else** S

E $\rightarrow$ **id**

Acertando a gramática

Símbolo inicial: **S**

Novo símbolo inicial: **S'**

$S' \rightarrow S \$$

$S \rightarrow \text{if } E \text{ then } S \ X$

$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

Acertando a gramática

Símbolo inicial: S

Novo símbolo inicial: S'

Nova regra de produção: $S' \rightarrow S \$$

$S' \rightarrow S \ \$$
 $S \rightarrow \text{if } E \text{ then } S \ X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow

Tabela de símbolos anuláveis, conjuntos first e conjuntos follow

$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'			
X			
S			
E			

Tabela de símbolos anuláveis, conjuntos first e conjuntos follow

$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×		
X	×		
S	×		
E	×		

Encontrando os símbolos não terminais anuláveis

Inicialmente nenhum símbolo não terminal é anulável

$S' \rightarrow S \ \$$

$S \rightarrow \text{if } E \text{ then } S \ X$

$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×		
X	×		
S	×		
E	×		

Regra com terminal no lado direito é inútil

$S' \rightarrow S \$$

$S \rightarrow \text{if } E \text{ then } S X$

$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×		
X	×		
S	×		
E	×		

Regra com terminal no lado direito é inútil

$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×		
X	×		
S	×		
E	×		

Regra com terminal no lado direito é inútil

$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×		
X	×		
S	×		
E	×		

Regra com terminal no lado direito é inútil

$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×		
X	×		
S	×		
E	×		

Regra com terminal no lado direito é inútil

$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×		
X	×	✓	
S	×		
E	×		

Lado direito da regra é vazio

$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×		
X	×	✓	
S	×		
E	×		

Encontrando os conjuntos first

Inicialmente são vazios

$S' \rightarrow S \$$

$S \rightarrow \text{if } E \text{ then } S \ X$

$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×		
X	×	✓	
S	×	if	
E	×		



símbolo terminal

$$\text{FIRST}(S) = \{ \} \cup \{ \text{if} \}$$

$S' \rightarrow S \$$

$S \rightarrow \text{if } E \text{ then } S \ X$

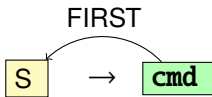
$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×		
X	×	✓	
S	×	if cmd	
E	×		



símbolo terminal

$$\text{FIRST}(S) = \{\text{if}\} \cup \{\text{cmd}\}$$

$S' \rightarrow S \$$

$S \rightarrow \text{if } E \text{ then } S \ X$

$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×		
X	×	✓	
S	×	if cmd	
E	×		

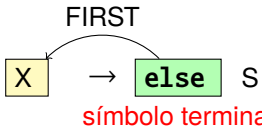
$X \rightarrow$

O lado direito da regra é ϵ

Logo a regra é inútil

$$S' \rightarrow S \quad \$$$
$$S \rightarrow \text{if } E \text{ then } S \text{ } X$$
$$S \rightarrow \text{cmd}$$
 $X \rightarrow$
$$X \rightarrow \text{else } S$$
$$E \rightarrow \mathbf{id}$$

NT	Nullable	First	Follow
S'	×		
X	×	else	
S	×	if cmd	
E	×		



$$\text{FIRST}(X) = \{ \} \cup \{ \text{else} \}$$

$S' \rightarrow S \$$

$S \rightarrow \text{if } E \text{ then } S X$

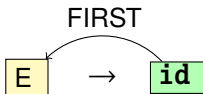
$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×		
X	×	else	
S	×	if cmd	
E	×	id	



símbolo terminal

$\text{FIRST}(E) = \{ \} \cup \{ \text{id} \}$

$S' \rightarrow S \$$

$S \rightarrow \text{if } E \text{ then } S \text{ } X$

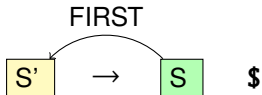
$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	else	
S	×	if cmd	
E	×	id	



símbolo não terminal

$$\text{FIRST}(S') = \{ \} \cup \{ \text{if}, \text{cmd} \}$$

$S' \rightarrow S \$$

$S \rightarrow \text{if } E \text{ then } S \ X$

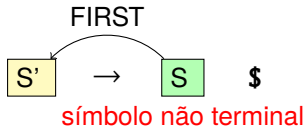
$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	else	
S	×	if cmd	
E	×	id	



$$\text{FIRST}(S') = \{\text{if}, \text{cmd}\} \cup \{\text{if}, \text{cmd}\}$$

$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S \mid X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	else	
S	×	if cmd	
E	×	id	

$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S \mid X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	else	
S	×	if cmd	
E	×	id	

$S' \rightarrow S \$$

$S \rightarrow \text{if } E \text{ then } S \ X$

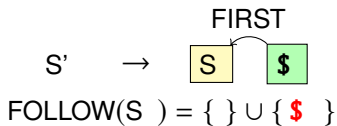
$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	else	
S	×	if cmd	\$
E	×	id	



$S' \rightarrow S \$$

$S \rightarrow \text{if } E \text{ then } S X$

$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	else	
S	×	if cmd	\$
E	×	id	then

FIRST

$S \rightarrow \text{if } E \text{ then } S X$

$\text{FOLLOW}(E) = \{ \} \cup \{ \text{then} \}$

$S' \rightarrow S \$$

$S \rightarrow \text{if } E \text{ then } S X$

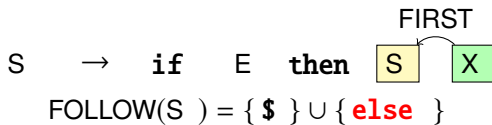
$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	else	
S	×	if cmd	\$ else
E	×	id	then



$S' \rightarrow S \$$

$S \rightarrow \text{if } E \text{ then } S X$

$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	else	
S	×	if cmd	\$ else
E	×	id	then

$S \rightarrow \text{if } E \text{ then } S X$

Não há nenhum símbolo após o último não terminal

$S' \rightarrow S \$$

$S \rightarrow \text{if } E \text{ then } S \ X$

$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	✓	else
S	×	if cmd	\$ else
E	×	id	then

$S \rightarrow \text{cmd}$

Nenhum símbolo não terminal no lado direito da regra

Regra inútil

$S' \rightarrow S \$$

$S \rightarrow \text{if } E \text{ then } S X$

$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	✓ else	
S	×	if cmd	\$ else
E	×	id	then

$X \rightarrow$

Nenhum símbolo não terminal no lado direito da regra

Regra inútil

$S' \rightarrow S \ \$$
 $S \rightarrow \text{if } E \text{ then } S \ X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	✓ else	
S	×	if cmd	\$ else
E	×	id	then

$X \rightarrow \text{else } S$

Não há nenhum símbolo após o último não terminal

$S' \rightarrow S \ \$$
 $S \rightarrow \text{if } E \text{ then } S \ X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	else	
S	×	if cmd	\$ else
E	×	id	then

$E \rightarrow \text{id}$

Nenhum símbolo não terminal no lado direito da regra

Regra inútil

$S' \rightarrow S \$$

$S \rightarrow \text{if } E \text{ then } S X$

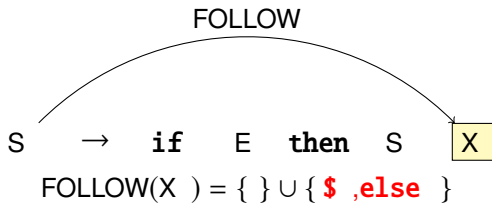
$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	else	\$ else
S	×	if cmd	\$ else
E	×	id	then



$S' \rightarrow S \$$

$S \rightarrow \text{if } E \text{ then } S X$

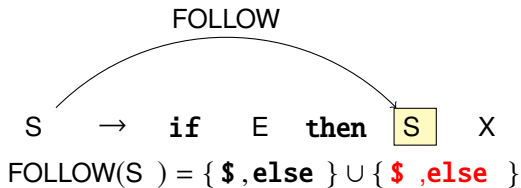
$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	else	\$ else
S	×	if cmd	\$ else
E	×	id	then

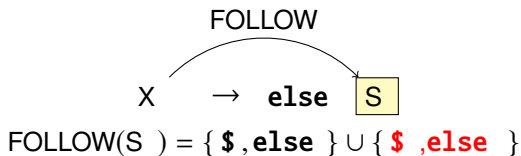


$$S' \rightarrow S \quad \$$$
$$S \rightarrow \text{if } E \text{ then } S \text{ X}$$
$$S \rightarrow \text{cmd}$$
 $X \rightarrow$

X → else S

$$E \rightarrow id$$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	else	\$ else
S	×	if cmd	\$ else
E	×	id	then



$S' \rightarrow S \$$

$S \rightarrow \text{if } E \text{ then } S X$

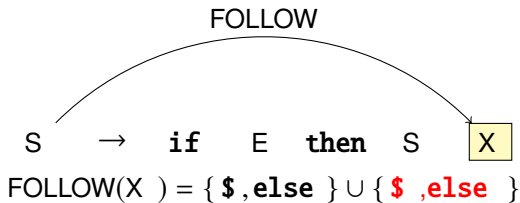
$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	else	$\$$ else
S	×	if cmd	$\$$ else
E	×	id	then



$S' \rightarrow S \$$

$S \rightarrow \text{if } E \text{ then } S X$

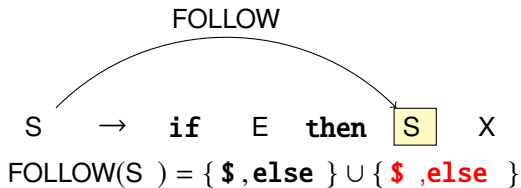
$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

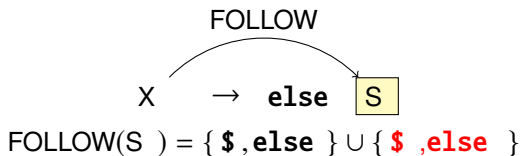
$E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	× ✓	else	\$ else
S	×	if cmd	\$ else
E	×	id	then



$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	else	\$ else
S	×	if cmd	\$ else
E	×	id	then



$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S \ X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	✓ else	\$ else
S	×	if cmd	\$ else
E	×	id	then

	\$	cmd	else	id	if	then
S'						
X						
S						
E						

Construção da tabela LL(1)

$S' \rightarrow S \$$

$S \rightarrow \text{if } E \text{ then } S \text{ } X$

$S \rightarrow \text{cmd}$

$X \rightarrow$

$X \rightarrow \text{else } S$

$E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	✓ else	\$ else
S	×	if cmd	\$ else
E	×	id	then

	\$	cmd	else	id	if	then
S'						
X						
S						
E						

$$\text{FIRST}(S \$) = \{ \} \cup \{ \text{if cmd} \}$$

$$S' \rightarrow S \quad \$$$
$$S \rightarrow \text{if } E \text{ then } S \text{ X}$$
$$S \rightarrow \text{cmd}$$
$$X \rightarrow$$
$$X \rightarrow \text{else } S$$
$$E \rightarrow \mathbf{id}$$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	else	\$ else
S	×	if cmd	\$ else
E	×	id	then

	\$	cmd	else	id	if	then
S'		S \$			S \$	
X						
S						
E						

$$S' \rightarrow S \quad \$$$

$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	✓ else	\$ else
S	×	if cmd	\$ else
E	×	id	then

	\$	cmd	else	id	if	then
S'		S \$			S \$	
X						
S						
E						

$$\text{FIRST}(\text{if } E \text{ then } S X) = \{ \} \cup \{ \text{if} \}$$

$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S \ X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	× ✓	else	\$ else
S	×	if cmd	\$ else
E	×	id	then

	\$	cmd	else	id	if	then
S'		S \$			S \$	
X						
S					if E then S X	
E						

$S \rightarrow \text{if } E \text{ then } S \ X$
 if

$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S \ X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	× ✓	else	\$ else
S	×	if cmd	\$ else
E	×	id	then

	\$	cmd	else	id	if	then
S'		S \$			S \$	
X						
S					if E then S X	
E						

$$\text{FIRST}(\text{cmd}) = \{ \} \cup \{ \text{cmd} \}$$

$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S \ X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	×	✓ else	\$ else
S	×	if cmd	\$ else
E	×	id	then

	\$	cmd	else	id	if	then
S'		S \$			S \$	
X						
S		cmd			if E then S X	
E						

$S \rightarrow \text{cmd}$
 cmd

$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S \ X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	× ✓	else	\$ else
S	×	if cmd	\$ else
E	×	id	then

	\$	cmd	else	id	if	then
S'		S \$			S \$	
X	€		€			
S		cmd			if E then S X	
E						

$X \rightarrow$
 $\$ \text{ else}$

$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S \text{ } X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	× ✓	else	\$ else
S	×	if cmd	\$ else
E	×	id	then

	\$	cmd	else	id	if	then
S'		S \$			S \$	
X	€		€			
S		cmd			if E then S X	
E						

$$\text{FIRST}(\text{else } S) = \{ \} \cup \{ \text{else} \}$$

$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S \ X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	× ✓	else	\$ else
S	×	if cmd	\$ else
E	×	id	then

	\$	cmd	else	id	if	then
S'		S \$			S \$	
X	€		€ else S			
S		cmd			if E then S X	
E						

$X \rightarrow \text{else } S$
 else

$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S \ X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	× ✓	else	\$ else
S	×	if cmd	\$ else
E	×	id	then

	\$	cmd	else	id	if	then
S'		S \$			S \$	
X	∈		∈ else S			
S		cmd			if E then S X	
E						

$$\text{FIRST}(\text{id}) = \{ \} \cup \{ \text{id} \}$$

$S' \rightarrow S \$$
 $S \rightarrow \text{if } E \text{ then } S \text{ } X$
 $S \rightarrow \text{cmd}$
 $X \rightarrow$
 $X \rightarrow \text{else } S$
 $E \rightarrow \text{id}$

NT	Nullable	First	Follow
S'	×	if cmd	
X	× ✓	else	\$ else
S	×	if cmd	\$ else
E	×	id	then

	\$	cmd	else	id	if	then
S'		S \$			S \$	
X	€		€ else S			
S		cmd			if E then S X	
E				id		

$E \rightarrow \text{id}$
 id