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Lexicographic goal programming approach for a short-term mining planning problem

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ABSTRACT

This article addresses a short-term mining planning problem. There are four objectives to be minimized: the deviations in grades and ore proportion in particle size ranges of the plant goals, the deviation in the waste mass to achieve the stripping rate, and the number of truck trips between mining fronts and discharges. The problem was solved through the lexicographical goal programming (LGP) method, which generates solutions that can guarantee a more comprehensive analysis of the decision-making process. The LGP method was tested by using several scenarios of a Brazilian mining company. These scenarios differ in the number of excavators and the tolerances concerning meeting the plants' ore grades. In the results, the impact on the values of the other objectives is analysed of varying the number of excavators and the tolerances in the plants' grade targets.

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Short-term planning; open-pit mining; grade and size ranges; mixed-integer linear programming; lexicographic goal programming

1. Introduction

The planning of mining activities is divided into three periods: long, medium and short term.

The primary purpose of long-term mining planning is to determine the geometry of the final surface of a deposit to maximize the profit of the extracted blocks, a problem that is known as the ultimate pit limit (UPL).

Historically, the UPL problem has been solved based on flow maximization in graph theory (Lerchs and Grossmann 1965). Other approaches have been applied to solve the UPL problem, such as mixed-integer linear programming (MILP) (Ben-Awuah *et al.* 2016).

Techniques involving uncertainties in the values of the block grades and commodity prices (Chatterjee, Sethi, and Asad 2016) have also been applied using stochastic optimization.

After solving the UPL problem, it is necessary to sequence the blocks' extraction in periods defined by the planner, usually annually, a problem known as open-pit mine production scheduling.

Medium-term mining planning aims to analyse the main actions necessary to guarantee the mine's operation for a period of up to 10 years, such as determining which regions of the mine require a license for expansion and purchase of equipment.

In turn, short-term mining planning (STMP) requires quick decisions to guarantee the fulfilment of the ore supply negotiated in contracts with customers. For this purpose, it is necessary to achieve objectives to meet the tonnages and grades desired by the plants, mine the waste, and reduce operating costs. These problems can be treated in a single period to determine the short-term schedule (Blom, Pearce, and Stuckey 2014; Upadhyay and Askari-Nasab 2015).

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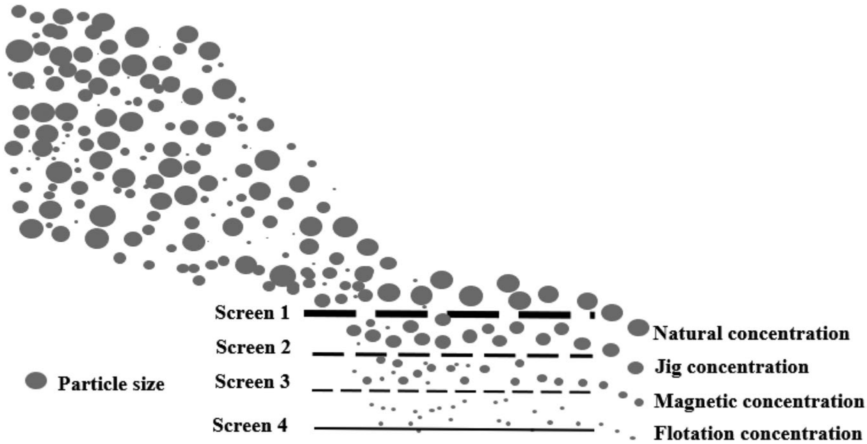

Figure 1. Ore classification according to particle size.

The STMP problem can also involve multiple periods, such as weeks or months, to establish the schedule, considering physical constraints in the sequence of block extraction (Eivazy and Askari-Nasab 2012; L'Heureux, Gamache, and Soumis 2013; Blom, Pearce, and Stuckey 2016, 2017). Additionally, these problems can address multiple sources and destinations of materials such as blocks, fronts, stockpiles, waste piles and plants, which can be located in various pits at the same mine.

Uncertainties caused by natural events or equipment breakdowns can be modelled using a discrete event simulator to validate the results of a short-term schedule provided by an optimizer (Fioroni *et al.* 2008; Upadhyay and Askari-Nasab 2018).

In some iron ore mines, one more factor increases the extraction complexity: the generation of products according to particle size. In this case, the ore plant concentrates the ore according to its particle size range. Figure 1 illustrates this case. Accordingly, the ore is concentrated through a specific method for each particle size range. For example, particles smaller than 0.15 mm are concentrated through the flotation process to generate pellet product feed. Therefore, it is necessary to control ore grades by particle size to produce ore with market specifications.

As shown later in the literature review section, few studies consider meeting the ore proportion in the particle size ranges required by plants. In plants in which mass and product quality depend on the particle size range and grade of the ore fed, it can be impossible to meet the values desired for these parameters. Therefore, it is essential to make a trade-off between mass production and product quality, especially in scenarios with few excavators available. Moreover, concerning the solution method, the most frequent approach is through a mixed-integer linear goal programming (MILGP) formulation. However, choosing the weights in the MILGP formulation is a complex task. In contrast, the lexicographic goal programming (LGP) method is an alternative to overcome this difficulty, as it generates solutions that can guarantee a more comprehensive analysis of the decision-making process. The LGP method assigns priorities to the different goals, minimizing deviations from the established goals in a lexicographic order (Romero 2001).

Therefore, the contributions of this work are as follows:

(1) the introduction of an LGP method to solve an STMP problem involving a work shift, multiple plants, waste piles, a heterogeneous fleet of trucks and excavators, and multiple pits in the same mine;
(2) an STMP with four goals to be minimized: (i) the deviations from the grade target of the chemical elements in the ore particle size ranges required by the plants, (ii) the deviations from the

ore proportion target in the particle size ranges required by the plants, (iii) the stripping ratio deviation, and (iv) the number of truck trips made between fronts and discharges; and
(3) an analysis of the impact on the values of the other objectives of varying the number of excavators and tolerances of the plants' grade targets.

The analysis concerning the third contribution is particularly important in deposits with wide variations in ore grades, since the greater the number of excavators, the greater the number of fronts that can be mined for meeting the required blending quality.

The remainder of this article is organized as follows. Section 2 presents a literature review. Section 3 describes and introduces the mathematical formulation for the problem addressed. Section 4 deals with the LGP method proposed for solving the STMP. In Section 5, the results of the LGP method in various scenarios of an iron ore mine are analysed and discussed. Finally, Section 6 outlines the final considerations.

2. Literature review

In the literature, STMP is treated through different approaches and can involve several goals and constraints. Table 1 presents its typical features, goals and constraints, while Table 2 shows the characteristics treated in STMP studies.

Pinto and Merschmann (2001) developed an MILP formulation that incorporates the capacity constraints of trucks and excavators into the model proposed by Chanda and Dagdelen (1995).

Costa, Souza, and Pinto (2004) developed an MILGP formulation that incorporates the meeting of the grade and production goals into the Pinto and Merschmann (2001) model. The authors also treated the trucks' haulage capacity depending on the cycle time and the payload.

Fioroni *et al.* (2008) incorporated into the model by Costa, Souza, and Pinto (2004) the objective of minimizing the movement of excavators between mining areas. The results of this model are used to perform the excavators' assignment and determine the number of trips each truck must make between mining fronts and discharge points. Based on the optimizer's results, a discrete event simulation model is then applied to treat stochastic occurrences that affect the efficiency of the equipment. The MILP formulation proposed in the present work is based on that of Fioroni *et al.* (2008). The differences between them are as follows:

Table 1. Typical features, goals and constraints treated in STMP problems.

Index	Features, goals and constraints
1	Plants' tonnage.
2	Plants' ore grades.
3	Plants' ore particle size range.
4	Compliance with stripping ratio.
5	Shovels' moving time between fronts.
6	Costs of resources or activities.
7	Number of trucks or trips.
8	Extraction of blocks with higher economic return.
9	Shovels' production
10	Long-term schedule compliance.
11	Equipment idle times.
12	Overall cost of mining operations.
13	Resource capacity.
14	Storage and reclaiming of ore stockpiles.
15	Precedence or other conditions for extracting blocks or fronts.
16	Restricting mining in specific regions.
17	Drilling and blasting of the blocks.
18	Multiperiod schedule.
19	Uncertainties in geological and/or operational data.

Table 2. Main features (✓), goals (G) and constraints (C) in STMP problems.

Study	Index according to Table 1																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Chanda and Dagdelen (1995)	G	G						G											
Pinto and Merschmann (2001)	G	C		C									C						
Costa, Souza, and Pinto (2004)	G	G		C				G					C						
Fioroni *et al.* (2008)	G	G		C	G								C						✓
Souza *et al.* (2010)	G	G		G			G		C				C						
Eivazy and Askari-Nasab (2012)	C	C		C						C		G	C	C	C			✓	
L'Heureux, Gamache, and Soumis (2013)	C	C		C	G	G			C				C		C		C	✓	
Upadhyay and Askari-Nasab (2015)	G	G		C	G	G			G				C						
Blom, Pearce, and Stuckey (2014)	G	G	G	C									C	C	C				
Blom, Pearce, and Stuckey (2016)	G	G	G	C									C	C	C			✓	
Matamoros and Dimitrakopoulos (2016)	G	G		G		G	C		G				C	G	G			✓	✓
Blom, Pearce, and Stuckey (2017)	C	G		G			G			G			C	G	C	G		✓	
Upadhyay and Askari-Nasab (2018)	G	G		C	G				G		G		C		C	C		✓	✓
Bakhtavar and Mahmoudi (2020)	G	C		C		G	G		G				C						✓
Flores-Fonseca, Linfati, and Escobar (2021)	C	C						G	G				C	C	C			✓	
This study	C	G	G	G			G		C				C			C			

(1) the LGP method to solve the mathematical model is applied and not the MILGP method;
(2) more than one type of ore from the mining fronts is included owing to the need to feed the dry plant with rich ore;
(3) multiple plants and waste piles are considered;
(4) the plants' mass targets are included as hard constraints and not as goals to be achieved;
(5) the minimization of the deviation of the ore proportion in the particle size range required by the plants is included as an objective since many iron ore plants have more than one process route depending on the required particle size range;
(6) the stripping ratio is included as an objective and not as a hard constraint to find feasible solutions for low-resource scenarios;
(7) the movement of excavators between the mining fronts is not considered;
(8) the impact of varying the number of excavators and tolerances of the plants' grade targets on the values of the other objectives is analysed.

In Souza *et al.* (2010), the authors incorporate into the MILGP method from Costa, Souza, and Pinto (2004) the minimization of the number of trucks needed for this process. They develop a hybrid heuristic algorithm that combines the characteristics of two meta-heuristics, greedy randomized adaptive search procedures and general variable neighbourhood search. However, the developed model only considers a single discharge.

Coelho *et al.* (2012) develop a multiobjective version of the problem addressed by Souza *et al.* (2010). They apply three multiobjective heuristic algorithms based on two-phase Pareto local search with variable neighbourhood search (2PPLS-VNS) (Lust, Teghem, and Tuyttens 2011), multiobjective VNS (MOVNS) (Geiger 2004; Duarte *et al.* 2015) and non-dominated sorting genetic algorithm II (NSGA-II) (Deb *et al.* 2002). They show that MOVNS outperforms the other algorithms concerning the hypervolume and spacing metrics.

Eivazy and Askari-Nasab (2012) apply an MILP formulation to obtain a multiperiod STMP. They also consider multiple destinations, selection of ramps, three horizontal mining directions of blocks, and routes to minimize the haulage costs.

L'Heureux, Gamache, and Soumis (2013) treat the STMP problem as multiperiod with physical constraints to extract blocks. To solve it, they propose an MILP method. However, the developed model cannot solve even an instance with ten periods, 30 faces, five shovels and 60 clusters of blocks for drilling and blasting. Therefore, they propose several strategies to reduce computation time.

Upadhyay and Askari-Nasab (2015) propose an MILGP model to work with a short-term production schedule, where faces are excavated within a given period of one month. Therefore, this model includes shovel assignment on the available faces, linking the tactical and strategic plans.

Matamoros and Dimitrakopoulos (2016) approach the SMTP problem through stochastic integer programming, considering uncertainties in metal grade, ore quality and equipment performance parameters.

Blom, Pearce, and Stuckey (2016) extend their previous work (Blom, Pearce, and Stuckey 2014), addressing the multiple periods, multiple mine planning problem (MTP-MMPP) of scheduling the production of multiple open-pit mines to supply ports with ore that can be blended to form products. In the short-term MTP-MMPP considered, a 13-week horizon is split into weekly periods. As do the present authors, these authors consider the particle size ranges of iron ore. However, unlike the present article, they treat compliance with the particle size ranges as a hard constraint.

Upadhyay and Askari-Nasab (2018) extend their previous model (Upadhyay and Askari-Nasab 2015) by including in its MILGP formulation an index to store the work shift (*i.e.* the period). Work shift information is used to keep track of mined faces over multiple periods. Additionally, a discrete event simulation model executes the optimizer results incorporating stochastic break events that reduce resource performance to validate the optimizer model's results. Therefore, the simulator can capture the uncertainties so that the planner can generate more realistic scenarios.

Kozan and Liu (2018) develop an MILP formulation for solving the SMTP problem in which the blocks must be drilled, blasted, and excavated up to the due date. There is a tardiness penalty in the objective function for blocks mined after the due date. They do not consider compliance with grades and particle size ranges.

Blom, Pearce, and Stuckey (2017) develop an LGP-based tool in which multiple, diverse, short-term schedules are constructed, seeking to meet a set of common objectives without the need for iterative parameter adjustment by the short-term planner. Multiple objectives are solved hierarchically, generating scenarios with different prioritizations of objectives. Unlike the present article, they do not consider the particle size ranges required by plants or the influence of the number of excavators.

Bakhtavar and Mahmoudi (2020) treat the STMP problem to perform truck–shovel allocation with uncertainties in shovel outputs, crusher capacity and the number of trucks. The authors formulate the truck–shovel allocation in two phases with concepts from a scenario-based robust model.

Flores-Fonseca, Linfati, and Escobar (2021) propose two MILP models that link strategic and operational problems. Both determine the sequencing and destination of the blocks; the first maximizes the net present value (NPV), and the second maximizes the work efficiency of the power shovels. With these models, it is possible to support decision-making and determine the optimal block extraction sequence in a given period with the maximum NPV of the project.

Unlike the proposal of this work, in none of the above references, except for Blom, Pearce, and Stuckey (2014, 2016), do the authors consider the target for ore proportion in particle size ranges. This information can be deduced from the survey carried out by Blom, Pearce, and Stuckey (2019), which reviews studies considered state-of-the-art for the STMP problem. In addition, no work has analysed the impact of the number of excavators on the results of other objectives and conducted a sensitivity analysis of the effect of increasing the range of grades tolerable by plants. Finally, only Blom, Pearce, and Stuckey (2017) treat the STMP through the LGP method, but not with the emphasis given to achieving the target of the proportion of ore in the particle size ranges required by plants.

3. Problem statement

Figure 2 illustrates the STMP scenario treated here. It shows some resources, locations and front materials involved in this problem. There is one excavator assigned to a front composed of four types of material (low-grade, middle-grade and high-grade iron ore and waste) and two types of truck (truck types 1 and 2) to transport these materials to two discharges—discharge 1 (plant) and discharge 5 (waste pile).

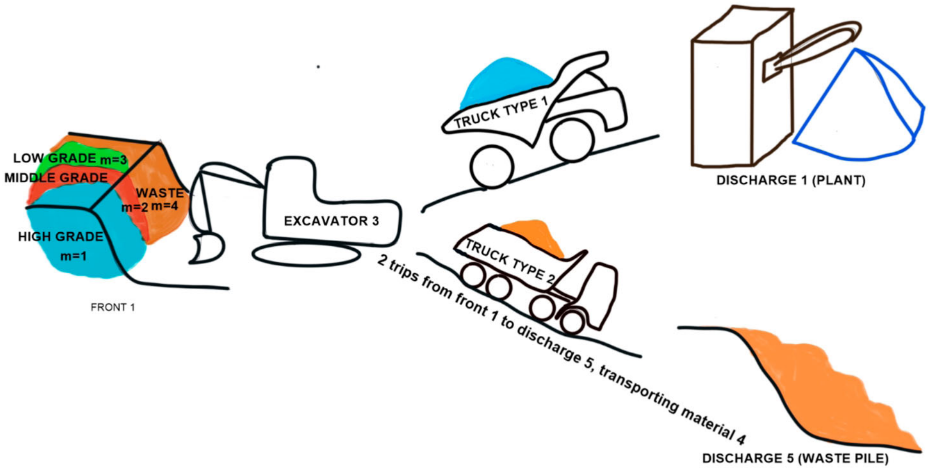

Figure 2. Example of an SMTP scenario.

3.1. Notation and definitions

This section describes the sets, indices, parameters and decision variables of the MILP formulation for the STMP problem.

3.1.1. Sets and indexes

$\mathcal{T}$ Set of truck types, indexed by t.
$\mathcal{E}$ Set of excavators, indexed by e.
$\mathcal{D}$ Set of discharges (plants or waste piles), indexed by k.
$\mathcal{F}$ Set of mining fronts, indexed by f.
$\mathcal{M}$ Set of material types by front, indexed by m.
$\mathcal{S}$ Set of particle size ranges, indexed by j.
$\mathcal{G}$ Set of chemical elements (Fe, Al_2O_3, Mn, *etc.*), indexed by i.

3.1.2. Parameters

DT_k A binary parameter that assumes a value of one if discharge $k \in \mathcal{D}$ is a plant and zero if it is a waste pile.
α Tolerance allowed to meet the production of plants with the available trucks in percent.
MM_{fm} Mass of material $m \in \mathcal{M}$ to be extracted from each front $f \in \mathcal{F}$ in tonnes.
ER_e Extraction rate of excavator $e \in \mathcal{E}$ in tonnes per hour.
SD Duration of the work shift, in hours (equal to 8 h in the present model).
MT_{fm} A binary parameter that assumes a value of one if material $m \in \mathcal{M}$ of front $f \in \mathcal{F}$ is ore and zero if it is waste.
TC_t Transportation capacity of each truck type $t \in \mathcal{T}$ in tonnes.
CT_{kfmt} Cycle time of truck type $t \in \mathcal{T}$ between front $f \in \mathcal{F}$ and discharge $k \in \mathcal{D}$ carrying material $m \in \mathcal{M}$ in minutes.
N_t Number of available trucks for each truck type $t \in \mathcal{T}$.
DR_k Production rate of ore discharge $k \in \mathcal{D}$ in tonnes per hour.
GT_{kji} Grade target of element $i \in \mathcal{G}$ in particle size range $j \in \mathcal{S}$ required by ore discharge $k \in \mathcal{D}$ in percent.
GM_{fmji} Grade of element $i \in \mathcal{G}$ in particle size range $j \in \mathcal{S}$ of material $m \in \mathcal{M}$ belonging to front $f \in \mathcal{F}$ in percent.
SP_{fmj} Percentage of particle size range $j \in \mathcal{S}$ of material $m \in \mathcal{M}$ belonging to front $f \in \mathcal{F}$.

ϵ Allowable tolerance from the grade target of element $i \in \mathcal{G}$ in particle size range $j \in \mathcal{S}$ required by ore discharge $k \in \mathcal{D}$ in percent.

ST_{kj} Target for the proportion of ore in particle size range $j \in \mathcal{S}$ required by ore discharge $k \in \mathcal{D}$ in percent.

WT Target for the stripping ratio.

NM Number of materials per front.

3.1.3. Decision variables

x_{fme} Binary variable that assumes a value of one if excavator $e \in \mathcal{E}$ is assigned to front $f \in \mathcal{F}$ to extract material $m \in \mathcal{M}$ and zero otherwise.

w_{kfmt} Number of trips of truck type $t \in \mathcal{T}$ between front $f \in \mathcal{F}$ and discharge $k \in \mathcal{D}$ to transport material $m \in \mathcal{M}$.

gd^+_{kji} Positive deviation from the grade target of element $i \in \mathcal{G}$ in particle size range $j \in \mathcal{S}$ required by ore discharge $k \in \mathcal{D}$ in tonnes.

gd^-_{kji} Negative deviation from the grade target of element $i \in \mathcal{G}$ in particle size range $j \in \mathcal{S}$ required by ore discharge $k \in \mathcal{D}$ in tonnes.

sd^+_{kj} Positive deviation from the mass target of particle size range $j \in \mathcal{S}$ required by ore discharge $k \in \mathcal{D}$ in tonnes.

sd^-_{kj} Negative deviation from the mass target of particle size range $j \in \mathcal{S}$ required by ore discharge $k \in \mathcal{D}$ in tonnes.

srd Deviation of the waste mass from the required stripping ratio WT in tonnes.

un_{kfmt} Fraction of the number of trucks of type $t \in \mathcal{T}$ used in each trip between front $f \in \mathcal{F}$ and discharge $k \in \mathcal{D}$ to transport material $m \in \mathcal{M}$.

3.2. Objective functions

The addressed problem has four objectives to be minimized.

The first objective, given by Equation (1), consists of minimizing the grade deviation of element $i \in \mathcal{G}$ within the particle size range $j \in \mathcal{S}$, in tonnes, from its grade target GT_{kji} required by ore discharge $k \in D$:

$$z_1 = \sum_{k \in \mathcal{D} | DT_k = 1} \sum_{j \in \mathcal{S}} \sum_{i \in \mathcal{G}} \left(gd^+_{kji} + gd^-_{kji} \right). \tag{1}$$

The second objective, given by Equation (2), minimizes the deviation of the particle size range $j \in \mathcal{S}$, in tonnes, from its target ST_{kj} required by ore discharge $k \in \mathcal{D}$:

$$z_2 = \sum_{k \in \mathcal{D} | DT_k = 1} \sum_{j \in \mathcal{S}} \left(sd^+_{kj} + sd^-_{kj} \right). \tag{2}$$

The third objective, given by Equation (3), aims at minimizing the deviation of the waste tonnage (srd) required to meet the stripping ratio target (WT). The variable srd and the parameter WT are used in Equation (17):

$$z_3 = srd. \tag{3}$$

The last objective, given by Equation (4), aims to minimize the number of trips of trucks made between the fronts and the discharges. This objective function prioritizes the use of trucks with larger payloads:

$$z_4 = \sum_{k \in \mathcal{D}} \sum_{f \in \mathcal{F}} \sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}} w_{kfmt}. \tag{4}$$

3.3. Constraints

There are seven groups of constraints, which are described in Sections 3.3.1–3.3.7.

3.3.1. Excavator assignment

Constraints (5) prevent each material on a front from being extracted by more than one excavator. In turn, Constraints (6) avoid the same excavator from extracting more than the number of materials per front. Constraints (7) ensure that the same excavator extracts all materials on the same front. Therefore, the value of variable x_{fme} must be the same for all materials $m \in \mathcal{M}$ present in one front f, thus requiring assigning the same excavator $e \in \mathcal{E}$ to the materials present in the same front $f \in \mathcal{F}$.

$$\sum_{e \in E} x_{fme} \leq 1 \quad \forall f \in \mathcal{F}, \forall m \in \mathcal{M} \tag{5}$$

$$\sum_{f \in \mathcal{F}} \sum_{m \in \mathcal{M}} x_{fme} \leq NM \quad \forall e \in \mathcal{E} \tag{6}$$

$$x_{fme} - x_{fte} = 0 \quad \forall e \in \mathcal{E}, \forall f \in \mathcal{F}, \forall m \in \mathcal{M}, \forall t \in \mathcal{M} \,|\, t \neq m. \tag{7}$$

3.3.2. Mass extraction

Constraints (8) establish that the mass extracted from each material m in front f to discharge k cannot exceed the existing material mass, expressed by the input data MM_{fm}. Constraints (9) ensure that the total mass extracted from the materials of each front cannot exceed the excavators' extraction capacity during the planning horizon. The extraction capacity of a material $m \in \mathcal{M}$ of each excavator $e \in \mathcal{E}$ assigned to front f is determined by the product between its productivity, in tonnes per hour (ER_e), and the duration of a work shift (SD):

$$\sum_{k \in \mathcal{D}} \sum_{t \in \mathcal{T}} TC_t \times w_{kfmt} \leq MM_{fm} \times \sum_{e \in \mathcal{E}} x_{fme} \quad \forall f \in \mathcal{F}, \forall m \in \mathcal{M} \tag{8}$$

$$\sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{D}} \sum_{t \in \mathcal{T}} TC_t \times w_{kfmt} \leq \frac{SD}{NM} \times \sum_{m \in \mathcal{M}} \sum_{e \in E} x_{fme} \times ER_e \quad \forall f \in \mathcal{F}. \tag{9}$$

3.3.3. Number of trips between fronts and discharges

Constraints (10) ensure that the maximum number of trips (w_{kfmt}) of the truck type $t \in \mathcal{T}$ between each front $f \in \mathcal{F}$ and each discharge $k \in \mathcal{D}$ to carry material $m \in \mathcal{M}$ depends on their cycle times CT_{kfmt} and the fraction un_{kfmt} of the number of trucks of this type over the shift duration SD. Constraints (11) ensure that the number of trucks per trip (un_{kfmt}) to carry materials $m \in \mathcal{M}$ from all fronts $f \in \mathcal{F}$ to all discharges $k \in \mathcal{D}$ must be less than the maximum number of trucks per fleet $t \in \mathcal{T}$:

$$w_{kfmt} \leq \sum_{k \in \mathcal{D}} \left(\left(\frac{60}{CT_{kfmt}} \right) \times un_{kfmt} \times SD \right) \quad \forall k \in \mathcal{D}, \forall f \in \mathcal{F}, \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \tag{10}$$

$$\sum_{f \in \mathcal{F}} \sum_{m \in \mathcal{M}} \sum_{k \in D} un_{kfmt} \leq N_t \quad \forall t \in \mathcal{T}. \tag{11}$$

3.3.4. Plants' capacity

Constraints (12) and (13) guarantee that the ore demanded by each plant k in a work shift is attended with α% tolerance from its capacity DR_k. This tolerance is necessary to ensure that the mass of ore sent by the trucks to the plants is a multiple of trucks' capacities:

$$\sum_{f \in \mathcal{F}} \sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}} TC_t \times w_{kfmt} \times MT_{fm} \geq (1 - \alpha) \times DR_k \times SD \quad \forall k \in \mathcal{D} \,|\, DT_k = 1 \tag{12}$$

$$\sum_{f\in\mathcal{F}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} TC_t \times w_{kfmt} \times MT_{fm} \leq (1+\alpha) \times DR_k \times SD \quad \forall\, k \in \mathcal{D}\,|\, DT_k = 1. \tag{13}$$

3.3.5. Grade

Constraints (14) allow for the mass of an element in a given particle size range extracted from the materials of all fronts destined for each ore discharge to exceed ϵ% of the grade target GT_{kji} established for this element in the respective plant. The tonnage of this element that exceeds this target is the value of the positive deviation gd^{+}_{kji}, which should be minimized in the objective function (1). Similarly, Constraints (15) allow for the mass of an element in a given particle size range to be less than ϵ% of its grade target. The lacking mass of this element is the value of the negative deviation gd^{-}_{kji}, which should be minimized in the objective function (1).

The purpose of these constraints is to set a lower and an upper bound to the grade of an element in a given particle size range required by the plants. That is, Constraints (14) set an upper bound for the grade in each particle size range, given by $(1+\epsilon) \times GT_{kji}$, while Constraints (15) set a lower bound, given by $(1-\epsilon) \times GT_{kji}$:

$$\begin{aligned}&\sum_{f\in\mathcal{F}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} GM_{fmji} \times TC_t \times w_{kfmt} \times SP_{fmj} - gd^{+}_{kji}\\ &\quad \leq (1+\epsilon) \times GT_{kji} \times \sum_{f\in F}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} TC_t \times w_{kfmt} \times SP_{fmj} \quad \forall\, k \in \mathcal{D}, DT_k = 1,\ \forall\, j \in S,\ \forall\, i \in \mathcal{G}.\end{aligned} \tag{14}$$

$$\begin{aligned}&\sum_{f\in\mathcal{F}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} GM_{fmji} \times TC_t \times w_{kfmt} \times SP_{fmj} + gd^{-}_{kji}\\ &\quad \geq (1-\epsilon) \times GT_{kji} \times \sum_{f\in\mathcal{F}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} TC_t \times w_{kfmt} \times SP_{fmj} \quad \forall\, k \in \mathcal{D},\ DT_k = 1,\ \forall\, j \in S,\ \forall\, i \in \mathcal{G}.\end{aligned} \tag{15}$$

3.3.6. Particle size range

Constraints (16) ensure compliance with the proportion of ore in the particle size range $j \in \mathcal{S}$ required by plant $k \in \mathcal{D}$:

$$\begin{aligned}&\sum_{f\in F}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} SP_{fmj} \times TC_t \times w_{kfmt} - sd^{+}_{kj} + sd^{-}_{kj}\\ &\quad = ST_{kj} \times \sum_{f\in\mathcal{F}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} TC_t \times w_{kfmt} \quad \forall\, k \in \mathcal{D},\ DT_k = 1,\ \forall\, j \in S.\end{aligned} \tag{16}$$

3.3.7. Stripping ratio

The waste tonnage required to meet the stripping ratio WT is forced by Constraint (17). A deviation equal to *srd* is allowed; however, it should be minimized in the objective function (4):

$$\begin{aligned}&\sum_{f\in\mathcal{F}}\sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{D}}\sum_{t\in\mathcal{T}} \left(1 - MT_{fm}\right) \times TC_t \times w_{kfmt} + srd\\ &\quad = WT \times \sum_{k\in\mathcal{D}}\sum_{f\in\mathcal{F}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} MT_{fm} \times TC_t \times w_{kfmt}.\end{aligned} \tag{17}$$

4. LGP method

Now, the LGP method proposed for solving the STMP problem is described. The main objective is to achieve the plants' grades since the iron ore price is proportional to the required grade targets for the products of the plants. Therefore, the LGP method is appropriate for solving this problem, as it is possible to choose the priority order of the objectives. Algorithm 1 shows its pseudo-code.

Algorithm 1: LGP

Input : Vector of objective functions $Z = \left(z_1, z_2, \ldots, z_{|Z|}\right)$,
set TOL of tolerances ϵ to be used in Equations (14) and (15),
TL for solving each MILP model
Output: Set Sol of MILP solutions from TOL
$i \leftarrow 1$ {Index for each tolerance value $\epsilon_i \in TOL$}
$Sol \leftarrow \emptyset$ {Set of solutions returned by the LGP method}
foreach $\epsilon_i \in TOL$ **do**
  $l \leftarrow 1$ {Index for the objective functions}
  $C \leftarrow \emptyset$ {Set containing the values returned by the MILP solver}
  while $l \leq |Z|$ **do**
    $model \leftarrow$ MILP model generated having z_l as objective function ($l \in \{1, 2, \ldots, |Z|\}$), $\epsilon = \epsilon_i$ as the tolerance used in Equations (14) and (15), and constraints $z_k \leq c_k \forall k = 1, \ldots, l-1$added to the model
    $c_l \leftarrow$ MILP-Solver($model$, TL) {Value of the lth objective function returned by the MILP solver after TL seconds at most}
    $C \leftarrow C \cup \{c_l\}$ $l \leftarrow l + 1$
  end
  $s \leftarrow$ solution returned by the MILP solver concerning the tolerance ϵ_i
  $Sol \leftarrow Sol \cup \{s\}$
  $i \leftarrow i + 1$
end
Return Sol {Set of solutions for all tolerances $\epsilon_i \in TOL$}

According to Algorithm 1, initially, three entries are provided: vector Z containing the objective functions (four in this study), set TOL with the admissible values for the tolerances ϵ adopted in Equations (14) and (15) (in this study, $TOL = \{0, 0.01, 0.02, 0.03, 0.04, 0.05\}$), and run time TL for solving each MILP model. The output is the set Sol of solutions generated by the MILP solver.

5. Computational experiments

The LGP method was implemented by using the Gurobi solver (Gurobi 2021), version 8.1.1, with the standard configuration. The experiments were performed on a computer with an Intel® i7-8550U @ 1.80 GHz × 4, 16 GB of RAM, and a Windows 10 operating system.

The LGP model and all data input and are available in the GitHub repository at https://github.com/aldringm/Eng_Optim_2021.git.

Section 5.1 describes the characteristics of the scenarios used for testing the proposed model. In Section 5.2, the LGP results are reported.

5.1. Characteristics of the scenarios

The LGP method was tested by using data from an iron ore mine belonging to a Brazilian mining company. This mine consists of two pits, each with its own waste pile, two plants differentiated by production, iron grade and particle size range targets, two truck types and five excavators. Figure 3 shows the mine layout.

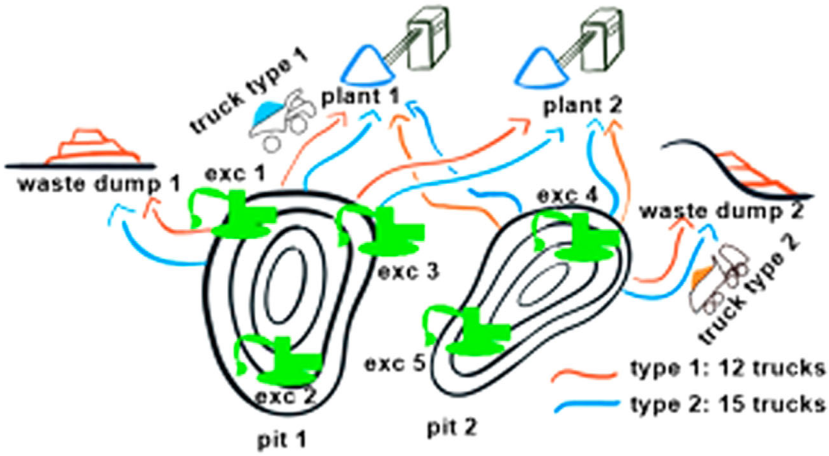

Figure 3. Layout of the mine.

Table 3. Description of particle size ranges.

Index j	Particle size range (mm)	Concentration process	Product
1	[6.3, 50]	Natural	Granulated
2	[0.5, 6.3]	Natural	Sinter feed
3	[0, 0.5]	Magnetic	Pellet feed

Table 4. Classification of material types.

Material	Iron grade (%)
Waste	≤ 42
Low-grade iron ore	> 42 and ≤ 45
Middle-grade iron ore	> 45 and ≤ 52
High-grade iron ore	> 52

In Figure 3, the arrows indicate the possible destinations of the materials. Therefore, ore from two pits can be destined for the two plants. In turn, each pit has its waste pile to receive the waste. Two types of truck are used, and five excavators for loading these trucks are used.

Table 3 describes the particle size ranges considered, the concentration process and the type of product generated.

Each front f has up to four types of material: waste ($m = 1$), low-grade iron ore ($m = 2$), middle-grade iron ore ($m = 3$) and high-grade iron ore ($m = 4$). They are classified by iron grade ranges according to Table 4.

Table 5 shows an example from a front. In this table, Column 1 shows the front index, Column 2 shows the type of material according to Table 4 and Column 3 shows the pit index. Column 4 presents a binary value that assumes a value of one if the material is ore and zero if it is waste. Column 5 shows the mass, in tonnes, of the corresponding material. Columns 6–8 show the ore proportion in the particle size range $j \in S$ of the material. Columns 9–11 present the iron grade in the particle size range $j \in S$. The last column presents the average of all material grades.

Table 6 informs the main characteristics of the mine. The iron metal is analysed, *i.e.* $i = 1$ in the parameters GT_{kji} and GM_{fmji}, variable gd^{+}_{kji} and in the set $\mathcal{G}$.

In the mine studied, the cycle times of the trucks vary according to the origin (mining front), destination (plant or waste pile), type of excavator, type of truck and type of material (waste or ore). Thus, for the same origin and destination, the cycle time varies according to the type of material. This

Table 5. Data from a front.

Front (f)	Material (m)	Pit	Ore or waste (MT_{fm})	Mass (tonnes)	Size range (%)			Iron grade (%)			
					1	2	3	1	2	3	$\bar{g}$
1	High-grade	1	1	5400	15	25	60	61	60	57	58.4
1	Middle-grade	1	1	7230	30	12	58	55	50	47	49.8
1	Low-grade	1	1	0	0	0	0	0	0	0	0
1	Waste	1	0	1690	20	14	66	38	36	27	30.5

Table 6. Main characteristics of the mine.

Characteristic	Value
Number of pits	2
Number of fronts	9 (1 to 6 at pit 1, and 7 to 9 at pit 2)
Number of truck fleets	2
Number of trucks per type	12 for fleet 1 and 15 for fleet 2
Truck capacity (tonnes)	135 for fleet 1 and 64 for fleet 2
Number of plants	2
Plants' productivity (tonnes per hour)	2350 and 275, respectively
α	0.01
Number of waste dumps	2, being one for each pit
Maximum number of excavators	5
Excavator extraction rate (tonnes per hour)	1200, 1500, 1300, 1300, 450
Shift time (hours)	8
Target for the stripping rate	0.78
Targets for plant 1:	
Ore proportion in the particle size range (%)	$ST_{11} = 23, ST_{12} = 26, ST_{13} = 50$
Iron grade targets (%)	$GT_{111} = 59, GT_{121} = 62, GT_{131} = 46$
Targets for plant 2:	
Ore proportion in the particle size range (%)	$ST_{21} = 20, ST_{22} = 24, ST_{23} = 47$
Iron grade targets (%)	$GT_{211} = 60, GT_{221} = 62, GT_{231} = 62$

is justified because denser materials (*i.e.* ores) require a longer cycle time owing to the greater effort of the excavators to extract these materials and load them onto the trucks. This information is available in the mine's dispatch system.

Twenty-four scenarios concerning the mine described previously were analysed. Each scenario differs concerning the number of excavators and the tolerable range for the plants' grade target established for each particle size range. The characteristics of these scenarios and the values of the grade targets for the plants are available in the GitHub repository at https://github.com/aldringm/Eng_Optim_2021.git.

5.2. Computational results

Table 7 reports the results obtained by the LGP method in each scenario. A run time of one hour is set for each model within the LGP method (*i.e.* $TL = 3600$ seconds). Thus, as there are four goals, the method could consume up to four hours of processing.

In the scenarios that use the greatest number of resources (Scenarios 19–24), the number of variables is 1584 (225 binary, 504 integers and 856 continuous), while the number of constraints is 1643.

According to Table 7, all scenarios with two excavators (*i.e.* 1 to 6) had the highest values for the stripping ratio deviation. This result shows that two excavators are not enough to meet the stripping ratio target.

In almost all scenarios, the iron grade deviations were null. The exception is for those with tolerance ϵ equal to zero and scenarios with two excavators.

Table 7. Scenarios' results. Optimal values are highlighted in bold.

Scenario	Excavators (number)	ϵ (%)	Run time (s)	Objective function value z_1	z_2	z_3	z_4	Trips (number) Fleet 1	Fleet 2	Plants' hourly tonnage Plant 1	Plant 2
1	2	0	2.33	**258.8**	**835.1**	**16,219.3**	**265**	54	211	2326.5	272.8
2	2	1	1.55	**149.1**	**739.0**	**16,219.3**	**265**	54	211	2326.5	272.8
3	2	2	1.35	**94.7**	**796.1**	**16,219.3**	**265**	54	211	2326.5	272.8
4	2	3	1.44	**63.4**	**841.4**	**16,219.3**	**265**	54	211	2326.5	272.8
5	2	4	1.70	**24.9**	**648.8**	**15,416.3**	**201**	123	78	2326.5	272.8
6	2	5	2.05	**1.4**	**1184.0**	**16,219.3**	**265**	54	211	2326.5	272.8
	Avg.		2.53	98.7	840.7	16085.4	253.8	65.5	188.8	2326.5	272.8
7	3	0	3.28	**37.6**	**5020.1**	**5019.3**	**369**	118	251	2326.5	272.8
8	3	1	1.48	**0.0**	**4395.7**	**5082.2**	**381**	107	274	2326.5	276.3
9	3	2	10.62	**0.0**	**1548.8**	**5056.5**	**314**	164	154	2326.5	276.3
10	3	3	595.51	**0.0**	**196.4**	**5114.9**	**301**	178	123	2326.5	272.8
11	3	4	814.12	**0.0**	**196.4**	**5024.1**	**339**	145	194	2326.6	272.8
12	3	5	7200.00	**0.0**	**196.4**	5016.1	348	137	211	2326.6	272.8
	Avg.		1437.51	6.2	1925.6	5532.6	341.9	135.5	206.6	2326.6	273.9
13	4	0	14.23	**12.9**	**5427.9**	**419.3**	**432**	126	306	2326.5	272.8
14	4	1	11.73	**0.0**	**3474.7**	**430.3**	**393**	161	232	2326.5	272.8
15	4	2	7200.00	**0.0**	**875.5**	434.4	423	134	289	2327.0	272.8
16	4	3	1619.20	**0.0**	**196.4**	**0.0**	**310**	244	66	2333.5	272.8
17	4	4	3604.36	**0.0**	**196.4**	**0.0**	309	245	64	2331.8	272.8
18	4	5	328.94	**0.0**	**196.4**	**0.0**	**298**	254	44	2326.5	272.8
	Avg.		2129.74	2.1	1727.9	214.0	360.8	194	166.8	2329.6	272.8
19	5	0	1.99	**12.9**	**5427.9**	**0.0**	**386**	175	211	2326.5	272.8
20	5	1	2.95	**0.0**	**3451.9**	**0.0**	**326**	228	98	2327.5	272.8
21	5	2	1.89	**0.0**	**875.6**	**0.0**	**325**	229	96	2326.6	272.8
22	5	3	640.48	**0.0**	**196.4**	**0.0**	**301**	253	48	2338.8	272.8
23	5	4	3839.08	**0.0**	**196.4**	**0.0**	296	257	39	2334.2	272.8
24	5	5	3645.50	**0.0**	**196.4**	0.0	**294**	258	36	2327.1	272.8
	Avg.		1355.31	2.1	1724.2	0.0	321.3	233.3	88	2330.1	272.8

In scenarios with three excavators (*i.e.* 7 to 12), the stripping ratio target deviations were approximately one-third of those with two excavators, indicating that it was possible to mine a part of the waste necessary with the addition of one excavator to fulfil the stripping ratio target.

The smallest deviations of the ore proportion in the particle size ranges occurred in scenarios with tolerance ϵ greater than or equal to 3%, except for those with two excavators.

The scenarios with four (*i.e.* 16 to 18) or five excavators (*i.e.* 22 to 24) and tolerance ϵ greater than or equal to 3% had the shortest values concerning the grade, ore proportion in particle size range and waste deviations.

Regarding the number of trips, it is only possible to compare the scenarios that obtained null deviations for the stripping ratio target or identical results for the total hauled masses. Therefore, when analysing Scenarios 16 to 24, it is noticed that the number of trips reduces as the tolerance ϵ increases. This occurs owing to the increase in the number of trips by the higher-capacity trucks (fleet 1), as shown in columns 9–10 of Table 7.

Regarding the computational time required by the LGP method, in all scenarios with two excavators or scenarios with a tolerance ϵ less than or equal to 2% (except the scenario with four excavators), the LGP method required less than 15 seconds to find the optimal solution for each objective. In Scenarios 12, 15, 17, 23 and 24, the LGP method did not find the optimal solution for the last two objective functions. As a result, the method required a computational time greater than 3600 seconds in these scenarios.

The analysis of all scenarios took approximately 8.2 hours, which indicates that the decision-maker should analyse all these scenarios at the start of the previous work shift. If a faster response is needed, then he/she should reduce the time limit imposed to run each model.

The graphical analysis of the iron grades for each particle size range, the ore proportion for each particle size range, and the stripping ratio are available in the GitHub repository of this work.

6. Conclusions

The complexity of mine operation depends on the number of resources available, the number of plants, their targets for the ore grade and proportion in the particle size range, waste piles, and mainly the daily volatility of mineral commodities' prices. Therefore, in times of crisis, or low market demand, reducing operating costs must be prioritized with minimal reduction in the ore's quality.

Usually, decision-makers at each stage of the mining companies' production chain are only interested in improving a particular stage's performance without analysing the impacts on the others. Therefore, this study showed the importance of making the plant's target grade more flexible and analysing the impact of small increments in the tolerance to these targets on the results of other objectives, especially in scenarios with a lower availability of excavators. As shown in the results, as the tolerance to the grade target increases, there is a reduction in deviations from the ore proportion targets in the particle size ranges. Therefore, making plant grade targets more flexible is essential, especially in plants that split the ore concentration according to the particle size range. This strategy is crucial, especially in situations with few excavators available.

In future work, it is intended to incorporate this proposal into a discrete event simulator to analyse the impact of these flexibilities on the production and quality of the products generated by the plant.

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Data availability statement

The data that support the findings of this study are openly available in the GitHub repository at https://github.com/aldringm/Eng_Opti

Disclosure statement

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