Step 1 Using the **Adjust** option of the menu, make the cell range B9..E11 your adjustable cells.

Step 2 Using the **Best** option and then the **Cost/Minimize** option select cell F2 as the objective function.

Step 3 Using the **Constrain** option select the cell range F9..F11 as the constraint range and use the **Less Than** option to constrain this range to be Less Than or Equal to the range H9..H11. After storing these constraints in the cell range G9..G11 we have created the supply constraints for the transportation problem.

Step 4 Using the **Constrain** option select the cell range B12..E12 and use the **Greater Than** option to constrain this range to be Greater Than or Equal to the cell range B14..E14. After storing these constraints in the cell range B13..E13 we have created the demand constraints.

Step 5 Use the **Solve** option to obtain the optimal solution in Figure 3(b).

From Figure 3(b), we see that What's Best has obtained the optimal solution to the Powerco problem given in Figure 1.

Problems

Group A

1 A company supplies goods to three customers, who each require 30 units. The company has two warehouses. Warehouse 1 has 40 units available and warehouse 2 has 30 units available. The costs of shipping 1 unit from warehouse to customer are shown in Table 7. There is a penalty for each unmet customer unit of demand: with customer 1 a penalty cost of $90 is incurred; with customer 2, $80; and with customer 3, $110. Formulate a balanced transportation problem to minimize the sum of shortage and shipping costs.

2 Referring to Problem 1, suppose that extra units could be purchased and shipped to either warehouse for a total cost of $100 per unit and that all customer demand must be met. Formulate a balanced transportation problem to minimize the sum of purchasing and shipping costs.

3 A shoe company forecasts the following demands during the next six months: month 1, 200; month 2, 260; month 3, 240; month 4, 340; month 5, 190; month 6, 150. It costs $7 to produce a pair of shoes with regular-time labor (RT) and $11 with overtime labor (OT). During each month, regular production is limited to 200 pairs of shoes, and overtime production is limited to 100 pairs. It costs $1 per month to hold a pair of shoes in inventory. Formulate a balanced transportation problem to minimize the total cost of meeting the next six months of demand on time.

4 Steelco manufactures three types of steel at different plants. The time required to manufacture 1 ton of steel (regardless of type) and the costs at each plant are shown in Table 8. Each week, 100 tons of each type of steel (1, 2, and 3) must be produced. Each plant is open 40 hours per week.

a Formulate a balanced transportation problem to minimize the cost of meeting Steelco's weekly requirements.

b Suppose the time required to produce 1 ton of steel depends on the type of steel as well as on the plant at which it is produced (see Table 9). Could a transportation problem still be formulated?

5 A hospital needs to purchase 3 gallons of a perishable medicine for use during the current month and 4 gallons for use during the next month. Because the medicine is perishable, it can only be used during the month of purchase. Two companies (Daisy and Laroach) sell the medicine. The medicine is in short supply. Thus, during the next two months, the hospital is limited to buying at most 5 gallons from each

TABLE 7

	To		
From	Customer 1	Customer 2	Customer 3
Warehouse 1	$15	$35	$25
Warehouse 2	$10	$50	$40

TABLE 8

	Cost			
	Steel 1	Steel 2	Steel 3	Time (minutes)
Plant 1	$60	$40	$28	20
Plant 2	$50	$30	$30	16
Plant 3	$43	$20	$20	15

TABLE 9

	Time (minutes)		
	Steel 1	Steel 2	Steel 3
Plant 1	15	12	15
Plant 2	15	15	20
Plant 3	10	10	15

company. The companies charge the prices shown in Table 10. Formulate a balanced transportation model to minimize the cost of purchasing the needed medicine.

6 A bank has two sites at which checks are processed. Site 1 can process 10,000 checks per day, and site 2 can process 6000 checks per day. The bank processes three types of checks: vendor, salary, and personal. The processing cost per check depends on the site (see Table 11). Each day, 5000 checks of each type must be processed. Formulate a balanced transportation problem to minimize the daily cost of processing checks.

7[†] The U.S. government is auctioning off oil leases at two sites: 1 and 2. At each site, 100,000 acres of land are to be auctioned. Cliff Ewing, Blake Barnes, and Alexis Pickens are bidding for the oil. Government rules state that no bidder can receive more than 40% of the land being auctioned. Cliff has bid \$1000/acre for site 1 land and \$2000/acre for site 2 land. Blake has bid \$900/acre for site 1 land and \$2200/acre for site 2 land. Alexis has bid \$1100/acre for site 1 land and \$1900/acre for site 2 land. Formulate a balanced transportation model to maximize the government's revenue.

8 The Ayatola Oil Company controls two oil fields. Field 1 can produce up to 40 million barrels of oil per day, and field 2 can produce up to 50 million barrels of oil per day. At field 1, it costs \$3 to extract and refine a barrel of oil; at field 2, the cost is \$2. Ayatola sells oil to two countries: England and Japan. The shipping cost per barrel is shown in Table 12. Each day, England is willing to buy up to 40 million barrels (at \$6 per barrel), and Japan is willing to buy up to 30 million barrels (at \$6.50 per barrel). Formulate a balanced transportation problem to maximize Ayatola's profits.

9 For the examples and problems of this section, discuss whether it is reasonable to assume that the proportionality assumption holds for the objective function.

10 Touche Young has three auditors. Each can work up to 160 hours during the next month, during which time three projects must be completed. Project 1 will take 130 hours, project 2, 140 hours, and project 3, 160 hours. The amount per hour that can be billed for assigning each auditor to each project is given in Table 13. Formulate a balanced transportation problem to maximize total billings during the next month.

Group B

11[‡] Paperco recycles newsprint, uncoated paper, and coated paper into recycled newsprint, recycled uncoated paper, and recycled coated paper. Recycled newsprint can be produced by processing newsprint or uncoated paper. Recycled coated paper can be produced by recycling any type of paper. Recycled uncoated paper can be produced by processing uncoated paper or coated paper. The process used to produce recycled newsprint removes 20% of the input's pulp, leaving 80% of the input's pulp for recycled paper. The process used to produce recycled coated paper removes 10% of the input's pulp. The process used to produce recycled uncoated paper removes 15% of the input's pulp. The purchasing costs, processing costs, and availability of each type of paper are shown in Table 14. To meet demand, Paperco must produce at least 250 tons of recycled newsprint pulp, at least 300 tons of recycled uncoated paper pulp, and at least 150 tons

TABLE 10

	Current Month's Price per Gallon	Next Month's Price per Gallon
Daisy	\$800	\$720
Laroach	\$710	\$750

TABLE 11

	Site 1	Site 2
Vendor checks	5¢	3¢
Salary checks	4¢	4¢
Personal checks	2¢	5¢

TABLE 12

	To	
From	England	Japan
Field 1	\$1	\$2
Field 2	\$2	\$1

TABLE 13

	Project		
Auditor	1	2	3
1	\$120	\$150	\$190
2	\$140	\$130	\$120
3	\$160	\$140	\$150

[†]This problem is based on Jackson (1980).

[‡]This problem is based on Glassey and Gupta (1974).

Problems

Group A

1 Five employees are available to perform four jobs. The time it takes each person to perform each job is given in Table 50. Determine the assignment of employees to jobs that minimizes the total time required to perform the four jobs.

2[†] Doc Councillman is putting together a relay team for the 400-meter relay. Each swimmer must swim 100 meters of breaststroke, backstroke, butterfly, or freestyle. Doc believes that each swimmer will attain the times given in Table 51. To minimize the team's time for the race, which swimmer should swim which stroke?

3 Tom Selleck, Burt Reynolds, Tony Geary, and John Travolta are marooned on a desert island with Olivia Newton-John, Loni Anderson, Dolly Parton, and Genie Francis. The "compatibility measures" in Table 52 indicate how much happiness each couple would experience if they spent all their time together. The happiness earned by a couple is proportional to the fraction of time they spend together. For example, if Tony and Genie spend half their time together, they earn happiness of $\frac{1}{2}(9) = 4.5$.

a Let x_{ij} be the fraction of time that the ith man spends with the jth woman. The goal of the eight people is to maximize the total happiness of the people on the island.

TABLE **50**

	Time (hours)			
	Job 1	Job 2	Job 3	Job 4
Person 1	22	18	30	18
Person 2	18	—	27	22
Person 3	26	20	28	28
Person 4	16	22	—	14
Person 5	21	—	25	28

Note: Dashes indicate person cannot do that particular job.

TABLE **51**

	Time (seconds)			
	Free	Breast	Fly	Back
Gary Hall	54	54	51	53
Mark Spitz	51	57	52	52
Jim Montgomery	50	53	54	56
Chet Jastremski	56	54	55	53

[†]This problem is based on Machol (1970).

TABLE **52**

	ONJ	LA	DP	GF
TS	7	5	8	2
BR	7	8	9	4
TG	3	5	7	9
JT	5	5	6	7

Formulate an LP whose optimal solution will yield the optimal values of the x_{ij}'s.

b Explain why the optimal solution in part (a) will have four $x_{ij} = 1$ and twelve $x_{ij} = 0$. Since the optimal solution requires that each person spend all his or her time with one person of the opposite sex, this result is often referred to as the Marriage Theorem.

c Determine the marriage partner for each person.

d Do you think the Proportionality Assumption of linear programming is valid in this situation?

4 A company is taking bids on four construction jobs. Three people have placed bids on the jobs. Their bid (in thousands of dollars) are given in Table 53 (a * indicates that the person did not bid on the given job). Person 1 can do only one job, but person 2 and person 3 can each do up to two jobs. Determine the minimum cost assignment of persons to jobs.

Group B

5 Any transportation problem can be formulated as an assignment problem. To illustrate the idea, determine an assignment problem that could be used to find the optimal solution to the transportation problem in Table 54. (*Hint:* You will need five supply and five demand points).

6 The Chicago Board of Education is taking bids on the city's four school bus routes. Four companies have made the bids in Table 55.

a Suppose each bidder can be assigned only one route. Use the assignment method to minimize Chicago's cost of running the four bus routes.

b Suppose that each company can be assigned two routes. Use the assignment method to minimize Chicago's cost of running the four bus routes. (*Hint:* Two supply points will be needed for each company.)

TABLE **53**

	Job			
	1	2	3	4
Person 1	50	46	42	40
Person 2	51	48	44	*
Person 3	*	47	45	45

TABLE 54

	3	1	2
	2	3	3
	1	4	

TABLE 55

	Bids			
	Route 1	Route 2	Route 3	Route 4
Company 1	\$4000	\$5000	—	—
Company 2	—	\$4000	—	\$4000
Company 3	\$3000	—	\$2000	—
Company 4	—	—	\$4000	\$5000

7 Show that Step 3 of the Hungarian method is equivalent to performing the following operations: (1) Add k to each cost that lies in a covered row. (2) Subtract k from each cost that lies in an uncovered column.

8 Suppose c_{ij} is the smallest cost in row i and column j of an assignment problem. Must $x_{ij} = 1$ in any optimal assignment?

7.6 Transshipment Problems

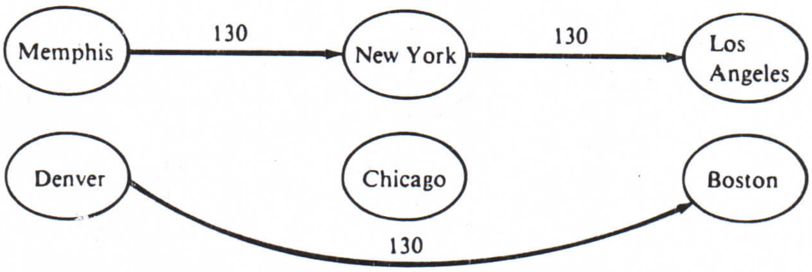
A transportation problem allows only shipments that go directly from a supply point to a demand point. In many situations, shipments are allowed between supply points or between demand points. Sometimes there may also be points (called transshipment points) through which goods can be transshipped on their journey from a supply point to a demand point. Shipping problems with any or all of these characteristics are transshipment problems. Fortunately, the optimal solution to a transshipment problem can be found by solving a transportation problem.

In what follows, we define a **supply point** to be a point that can send goods to another point but cannot receive goods from any other point. Similarly, a **demand point** is a point that can receive goods from other points but cannot send goods to any other point. A **transshipment point** is a point that can both receive goods from other points and send goods to other points. The following example illustrates these definitions ("—" indicates that a shipment is impossible).

EXAMPLE 5 Widgetco manufactures widgets at two factories, one in Memphis and one in Denver. The Memphis factory can produce up to 150 widgets per day, and the Denver factory can produce up to 200 widgets per day. Widgets are shipped by air to customers in Los Angeles and Boston. The customers in each city require 130 widgets per day. Because of the deregulation of air fares, Widgetco believes that it may be cheaper to first fly some widgets to New York or Chicago and then fly them to their final destinations. The costs of flying a widget are shown in Table 56. Widgetco wants to minimize the total cost of shipping the required widgets to its customers.

In this problem, Memphis and Denver are supply points, with supplies of 150 and 200 widgets per day, respectively. New York and Chicago are transshipment points. Los

FIGURE 9
Optimal Solution to Widgetco



shipments between demand points L.A. and Boston were allowed. This would make L.A. and Boston transshipment points, and add rows for L.A. and Boston. The supply for both the L.A. and Boston rows would be $0 + 350 = 350$. The demand for both the L.A. and Boston columns would now be $130 + 350 = 480$.

Problems

Group A

1 General Ford produces cars at L.A. and Detroit and has a warehouse in Atlanta; the company supplies cars to customers in Houston and Tampa. The cost of shipping a car between points is given in Table 58 ("—" means that a shipment is not allowed). L.A. can produce up to 1100 cars, and Detroit can produce up to 2900 cars. Houston must receive 2400 cars, and Tampa must receive 1500 cars.

a Formulate a balanced transportation problem that can be used to minimize the shipping costs incurred in meeting demands at Houston and Tampa.

b Modify the answer to part (a) if shipments between L.A. and Detroit are not allowed.

c Modify the answer to part (a) if shipments between Houston and Tampa are allowed at a cost of $5.

2 Sunco Oil produces oil at two wells. Well 1 can produce up to 150,000 barrels per day, and well 2 can produce up to 200,000 barrels per day. It is possible to ship oil directly from the wells to Sunco's customers in Los Angeles and New York. Alternatively, Sunco could transport oil to the ports of Mobile and Galveston and then ship it by tanker to New York or Los Angeles. Los Angeles requires 160,000 barrels per day, and New York requires 140,000 barrels per day. The costs of shipping 1000 barrels between two points are shown in Table 59. Formulate a transshipment model (and equivalent transportation model) that could be used to minimize the transport costs in meeting the oil demands of Los Angeles and New York.

3 In Problem 2, assume that before being shipped to Los Angeles or New York, all oil produced at the wells must be refined at either Galveston or Mobile. To refine 1000 barrels of oil costs $12 at Mobile and $10 at Galveston. Assuming that both Mobile and Galveston have infinite refinery capacity, formulate a transshipment and balanced transportation

TABLE 58

	To				
From	**L.A.**	**Detroit**	**Atlanta**	**Houston**	**Tampa**
L.A.	$0	$140	$100	$90	$225
Detroit	$145	$0	$11[illegible]	$110	$119
Atlanta	$105	$115	$0	$113	$78
Houston	$89	$109	$121	$0	—
Tampa	$210	$117	$82	—	$0

TABLE 59

	To					
From	**Well 1**	**Well 2**	**Mobile**	**Galveston**	**N.Y.**	**L.A.**
Well 1	$0	—	$10	$13	$25	$28
Well 2	—	$0	$15	$12	$26	$25
Mobile	—	—	$0	$6	$16	$17
Galveston	—	—	$6	$0	$14	$16
N.Y.	—	—	—	—	$0	$15
L.A.	—	—	—	—	$15	$0

Note: Dashes indicate shipments that are not allowed.

model to minimize the daily cost of transporting and refining the oil requirements of Los Angeles and New York.

4 Rework Problem 3 under the assumption that Galveston has a refinery capacity of 150,000 barrels per day and Mobile has one of 180,000 barrels per day. (*Hint:* Modify the method used to determine the supply and demand at each transshipment point to incorporate the refinery capacity restrictions, but make sure to keep the problem balanced.)

5 General Ford has two plants, two warehouses, and three customers. The locations of these are as follows:

Plants: Detroit and Atlanta

Warehouses: Denver and New York

Customers: Los Angeles, Chicago, and Philadelphia

Cars are produced at plants, then shipped to warehouses, and finally shipped to customers. Detroit can produce 150 cars per week and Atlanta can produce 100 cars per week. Los Angeles requires 80 cars per week, Chicago, 70; and Philadelphia, 60. It costs $10,000 to produce a car at each plant and the cost of shipping a car between two cities is given in Table 60. Determine how to meet General Ford's weekly demands at minimum cost.

TABLE 60

	To	
From	**Denver**	**New York**
Detroit	$1253	$637
Atlanta	$1398	$841

	To		
From	**Los Angeles**	**Chicago**	**Philadelphia**
Denver	$1059	$996	$1691
New York	$2786	$802	$100

Group B

6[†] A company must meet the following demands for cash at the beginning of each of the next six months: month 1, $200; month 2, $100; month 3, $50; month 4, $80; month 5, $160; month 6, $140. At the beginning of month 1, the company has $150 in cash and $200 worth of bond 1, $100 worth of bond 2, and $400 worth of bond 3. The company will have to sell some bonds to meet demands, but a penalty will be charged for any bonds sold before the end of month 6. The penalties for selling $1 worth of each bond are as shown in Table 61.

a Assuming that all bills must be paid on time, formulate a balanced transportation problem that can be used to minimize the cost of meeting the cash demands for the next six months.

TABLE 61

	Month of Sale					
Bond	**1**	**2**	**3**	**4**	**5**	**6**
1	$0.21	$0.19	$0.17	$0.13	$0.09	$0.05
2	$0.50	$0.50	$0.50	$0.33	$0	$0
3	$1.00	$1.00	$1.00	$1.00	$1.00	$0

b Assume that payment of bills can be made after they are due, but a penalty of 5¢ per month is assessed for each dollar of cash demands that is postponed for one month. Assuming all bills must be paid by the end of month 6, develop a transshipment model that can be used to minimize the cost of paying the next six months' bills. (*Hint:* Transshipment points are needed, in the form Ct = cash available at beginning of month t after bonds for month t have been sold, but before month t demand is met. Shipments into Ct occur from bond sales and $Ct - 1$. Shipments out of Ct occur to $Ct + 1$ and demands for months $1, 2, \ldots t$.)

Summary

Notation

m = number of supply points

n = number of demand points

x_{ij} = number of units shipped from supply point i to demand point j

[†]Based on Srinivasan (1974).