Automato com Pilha
Pushdown Automaton

Algoritmos Recursivos e Pilhas

Princípio Geral em Computação: Qualquer algoritmo recursivo pode ser transformado em um não-recursivo usando-se uma pilha e um while-loop, que termina apenas quando a pilha está vazia.

EX: JVM mantém os registros de ativação de cada chamada de método. Considere:

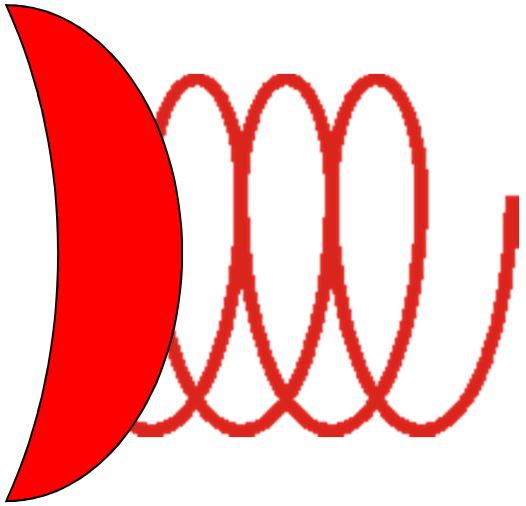
```
long factorial(int n){
  if (n<=0) return 1;
  return n*factorial(n-1);
}
```

Como JVM executa `factorial(5)`?

Algoritmos Recursivos e Pilhas

```
long factorial(int n){
   if (n<=0) return 1;
   return n*factorial(n-1);
}
```

Compute 5!



Algoritmos Recursivos e Pilhas

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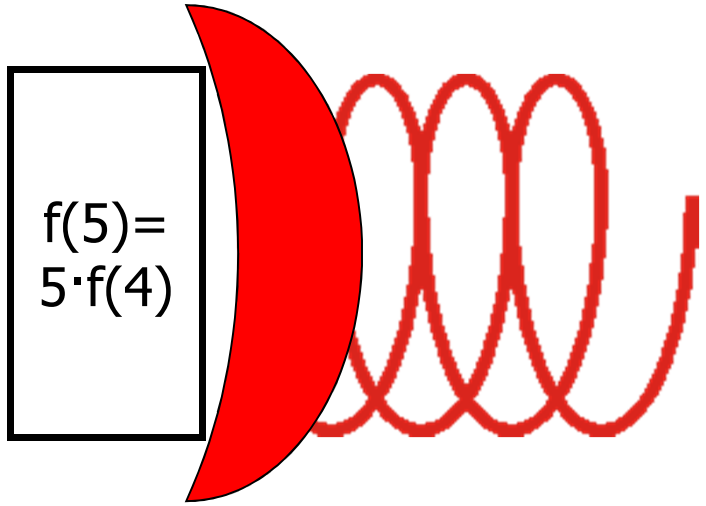

Algoritmos Recursivos e Pilhas

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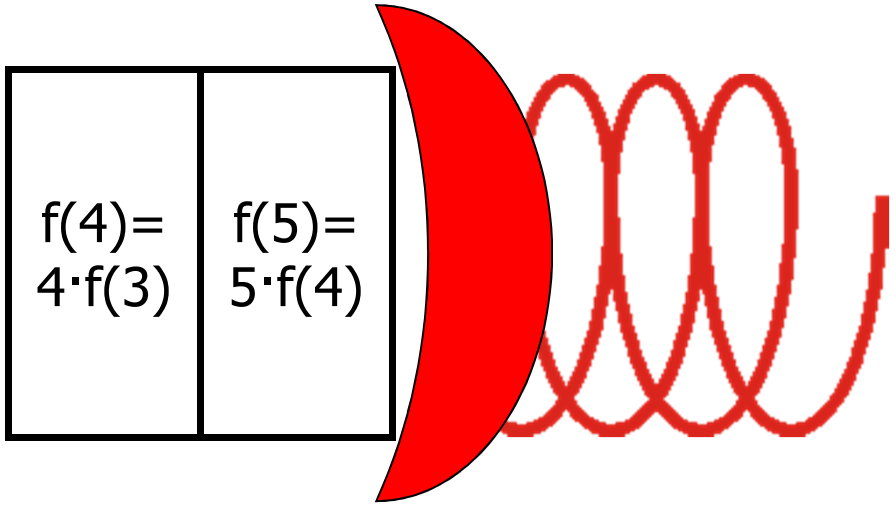

Algoritmos Recursivos e Pilhas

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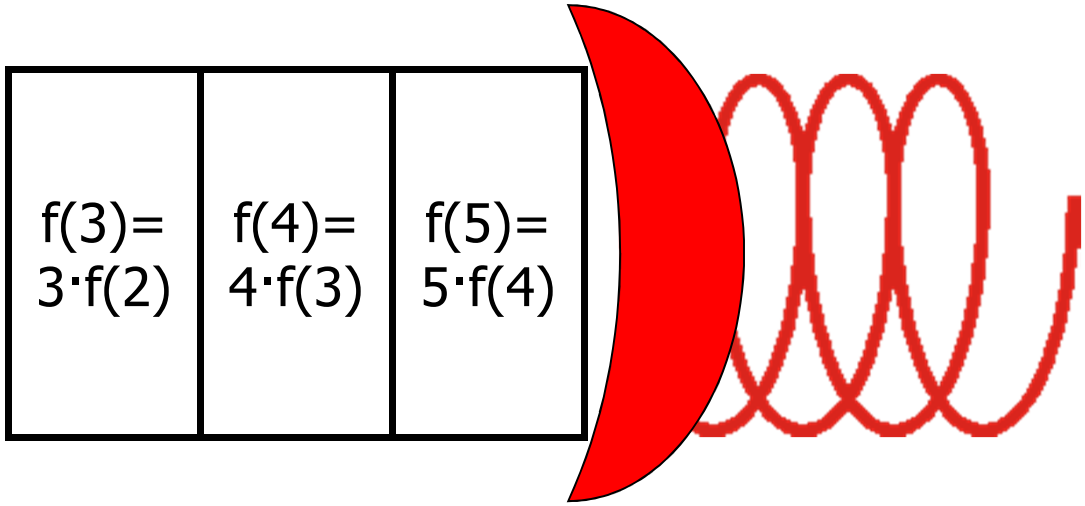

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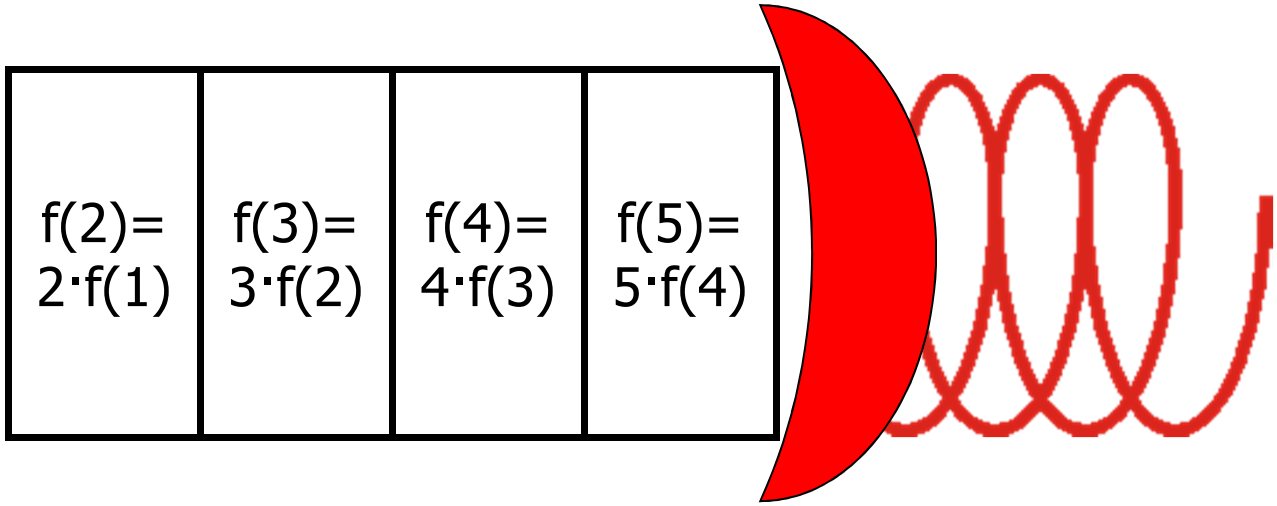

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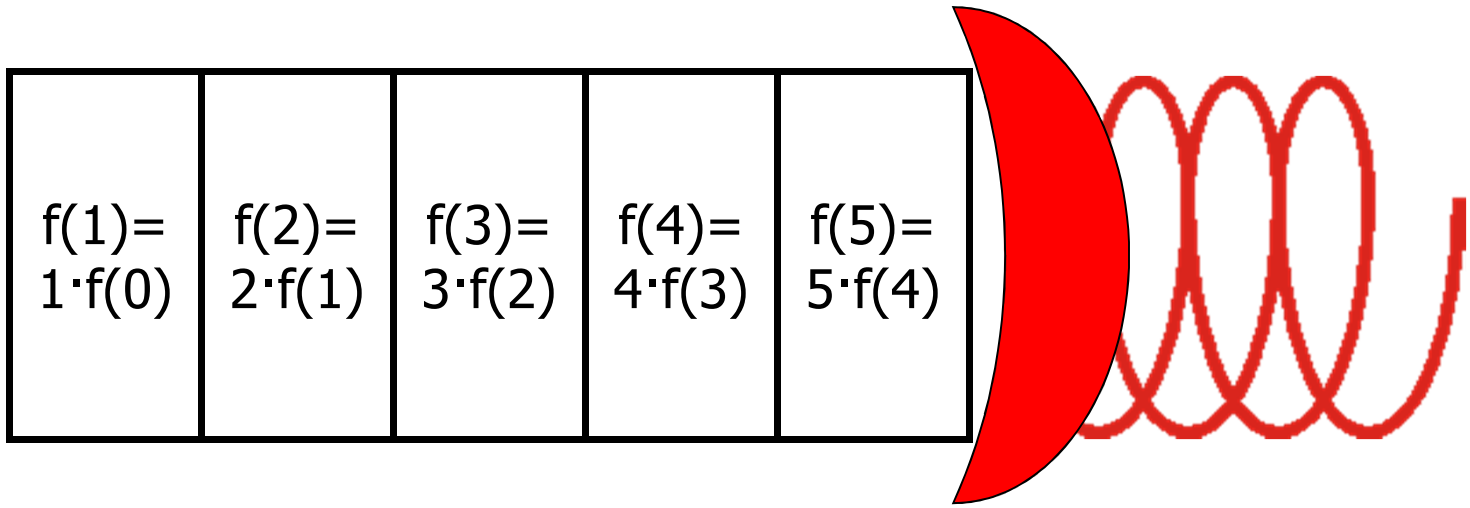

Algoritmos Recursivos e Pilhas

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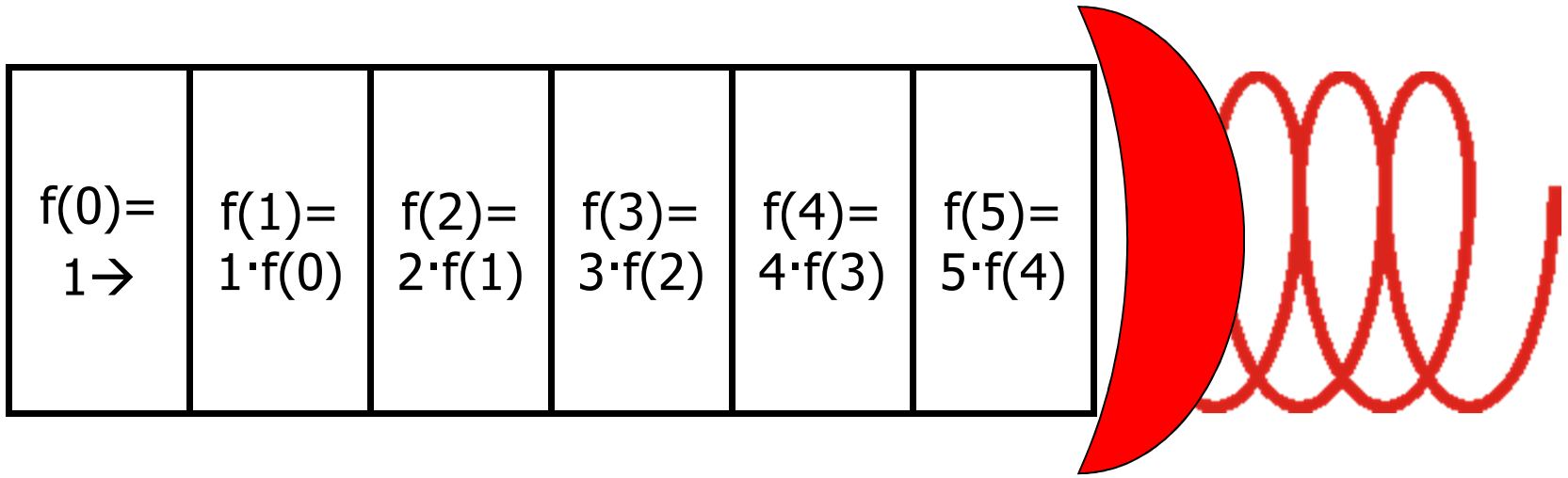

Algoritmos Recursivos e Pilhas

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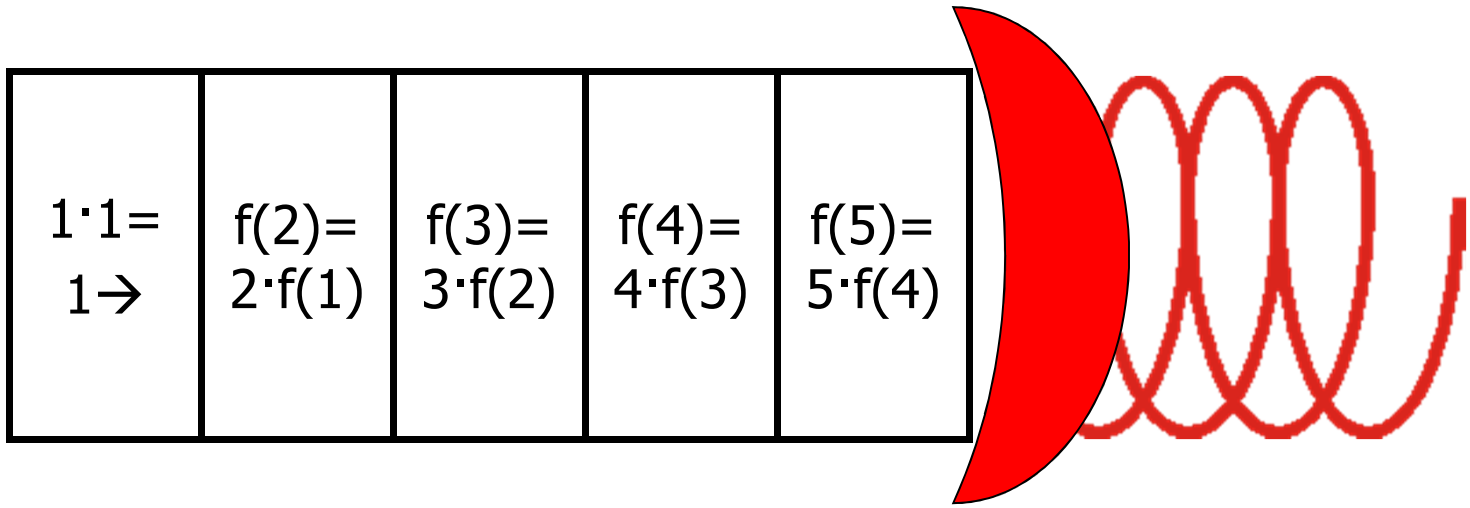

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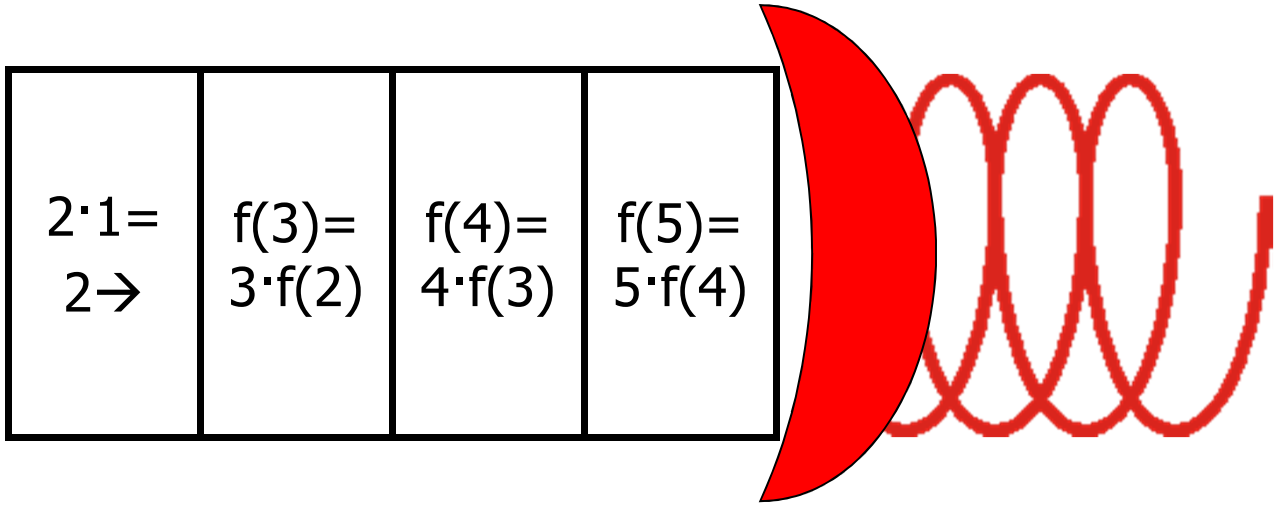

Algoritmos Recursivos e Pilhas

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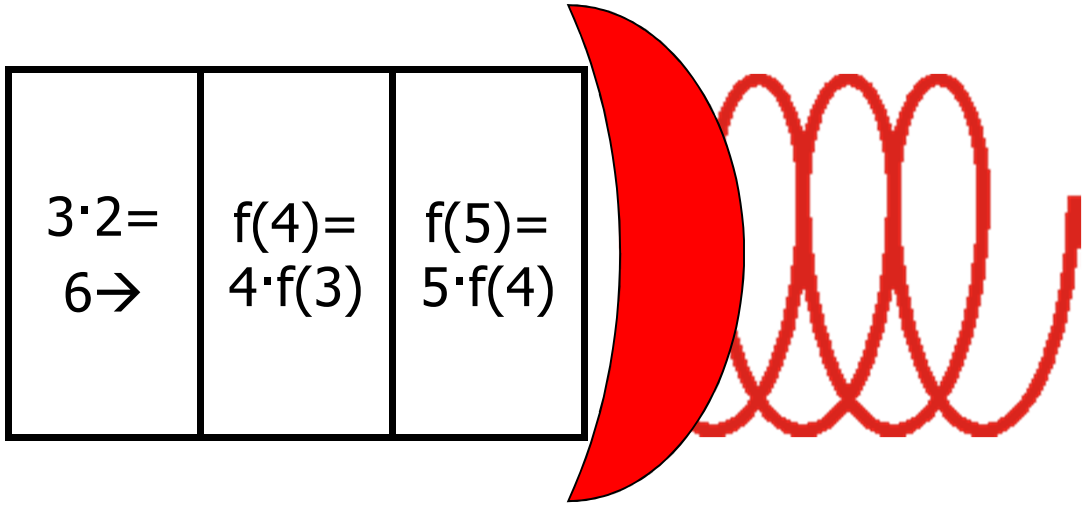

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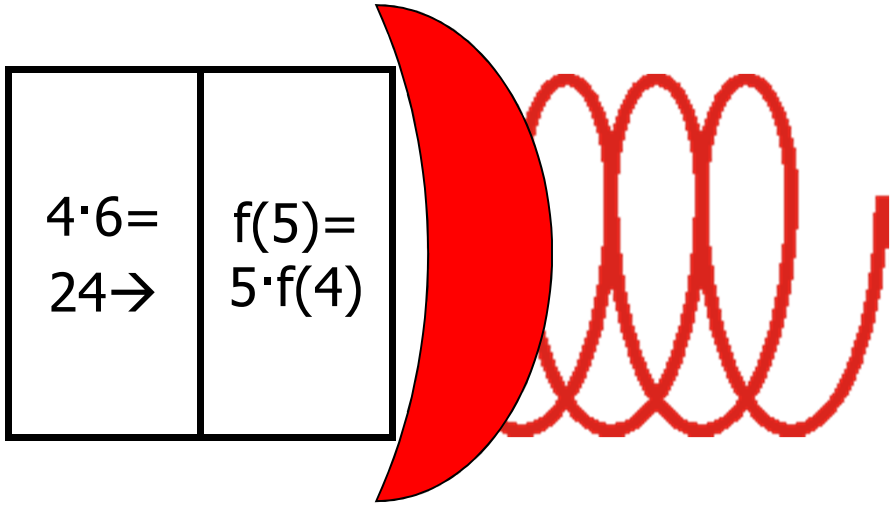

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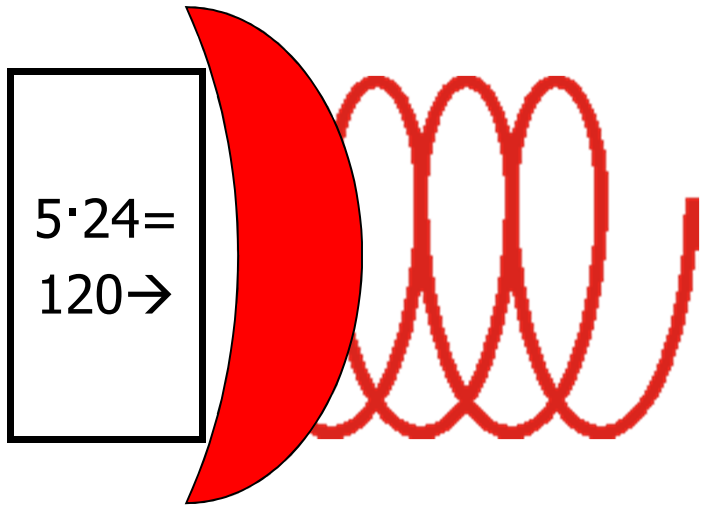

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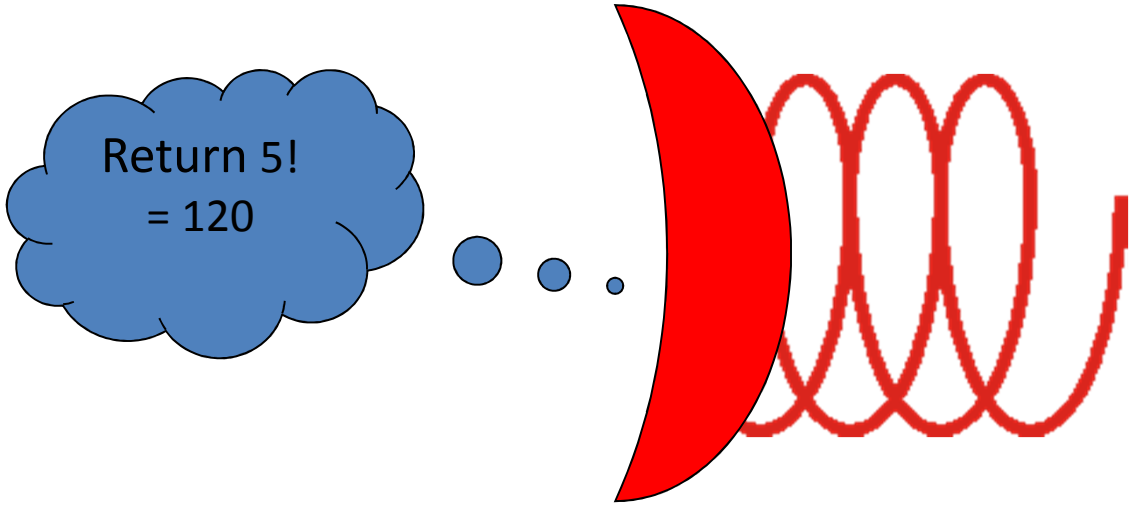


De CFG's para Autômatos com Pilha

CFG's definem procedimentos recursivos:

```
boolean derives(strings x, y)
1. if (x==y) return true
2. for(all u⇒y)
       if derives(x,u) return true
3. return false
```

EX: $S \rightarrow \# \mid aSa \mid bSb$

De CFG's para Máquinas de Pilha

Por princípios gerais, *qualquer* computação recursiva pode ser executada usando-se uma pilha. Isso pode ser feito em uma versão simplificada de máquina com pilha de registro de ativação, chamada Autômato de Pilha ("**P**ush**d**own **A**utomaton"– **PDA**)

Q: Qual é a linguagem gerada por

$S \rightarrow \# \mid aSa \mid bSb$?

De CFG's para Máquinas de Pilha

R: Palíndromos em {*a,b,*#}* contendo exatamente um símbolo #.

Q: Usando uma pilha, como podemos reconhecer tais strings?

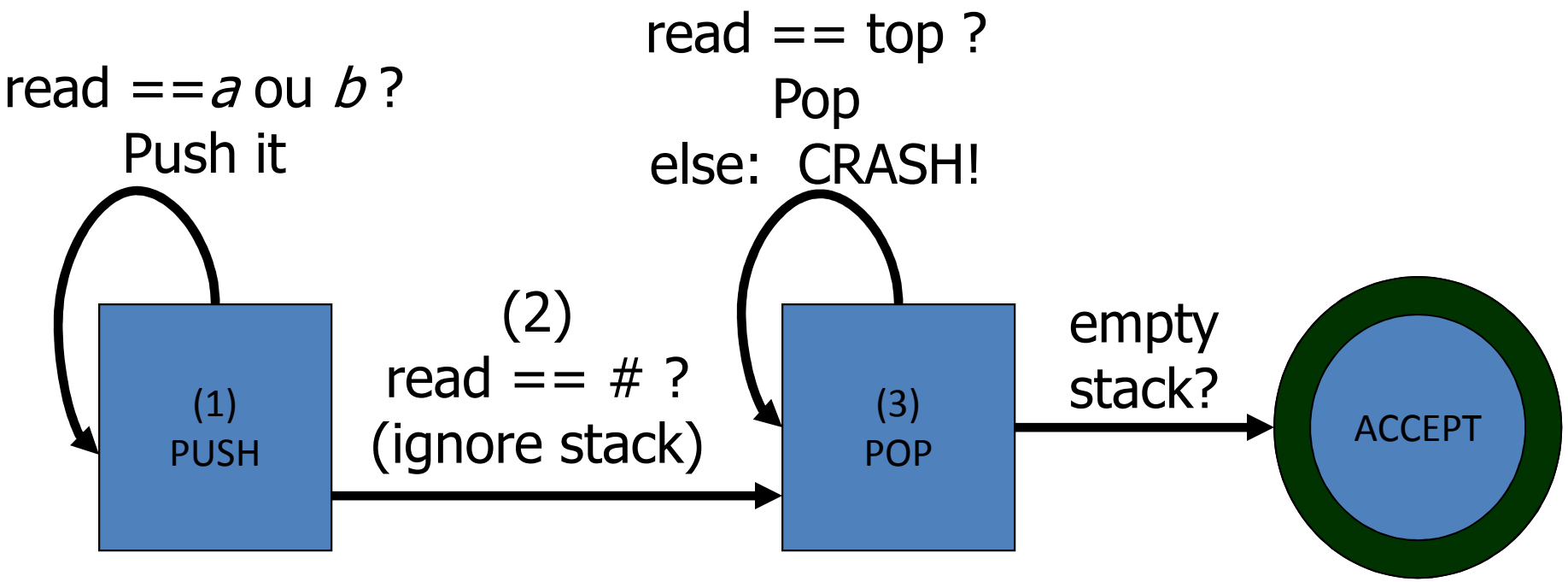
De CFG's para Máquinas de Pilha

A: Usamos um processo com três fases:

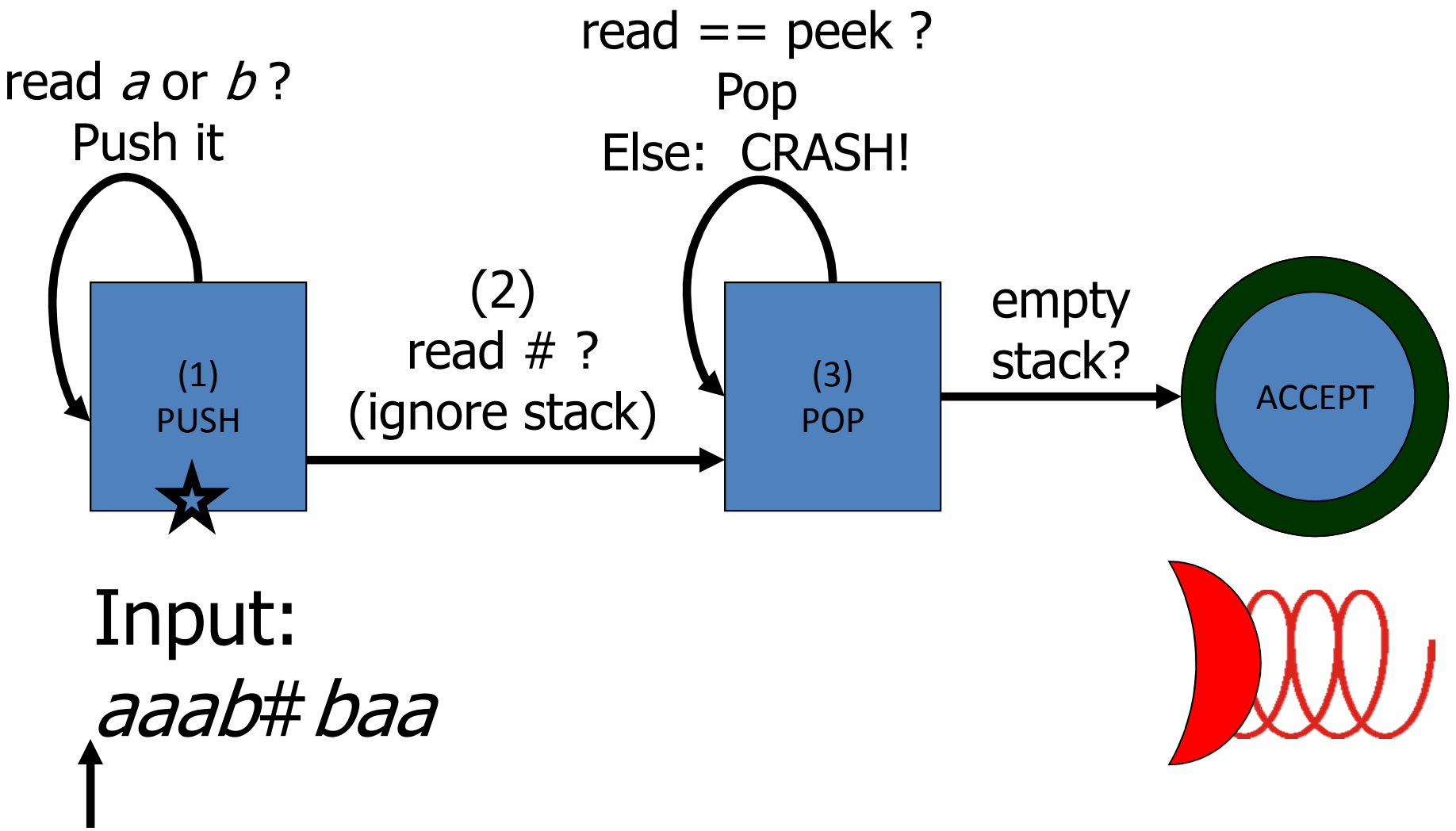
1. **modo Push :** Antes de ler "#", emplihe qualquer símbolo lido.
2. Ao ler "#" troque de modo de operação.
3. **modo Pop :** Leia os símbolos restantes garantindo que cada novo símbolo lido é idêntico ao símbolo desempilhado.

Aceite se for capaz de concluir com a pilha vazia. Caso contrário, rejeite; e rejeite se não for possível desempilhar em algum caso.

De CFG's para Máquinas de Pilha



De CFG's para Máquinas de Pilha



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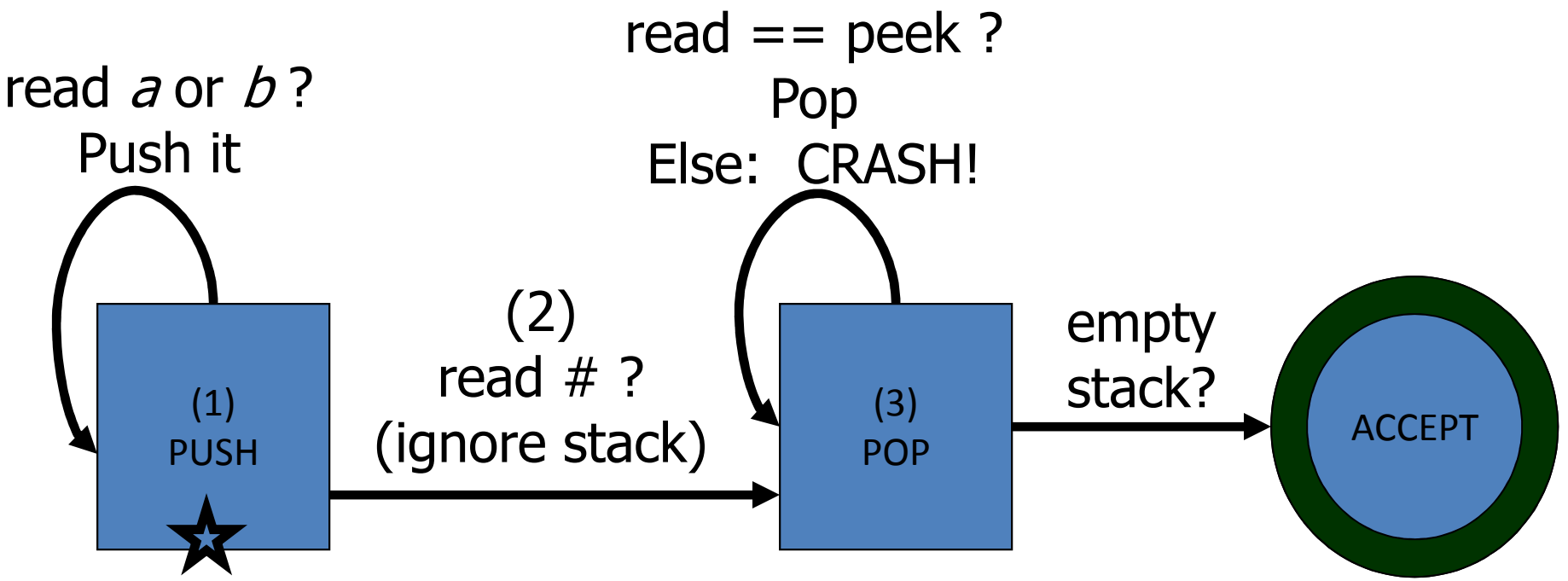
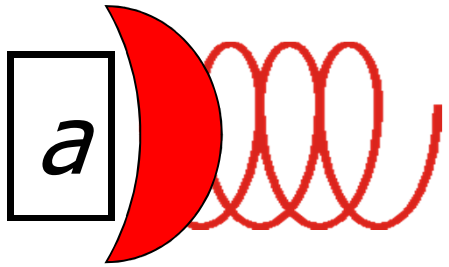


Input:

aaab# baa



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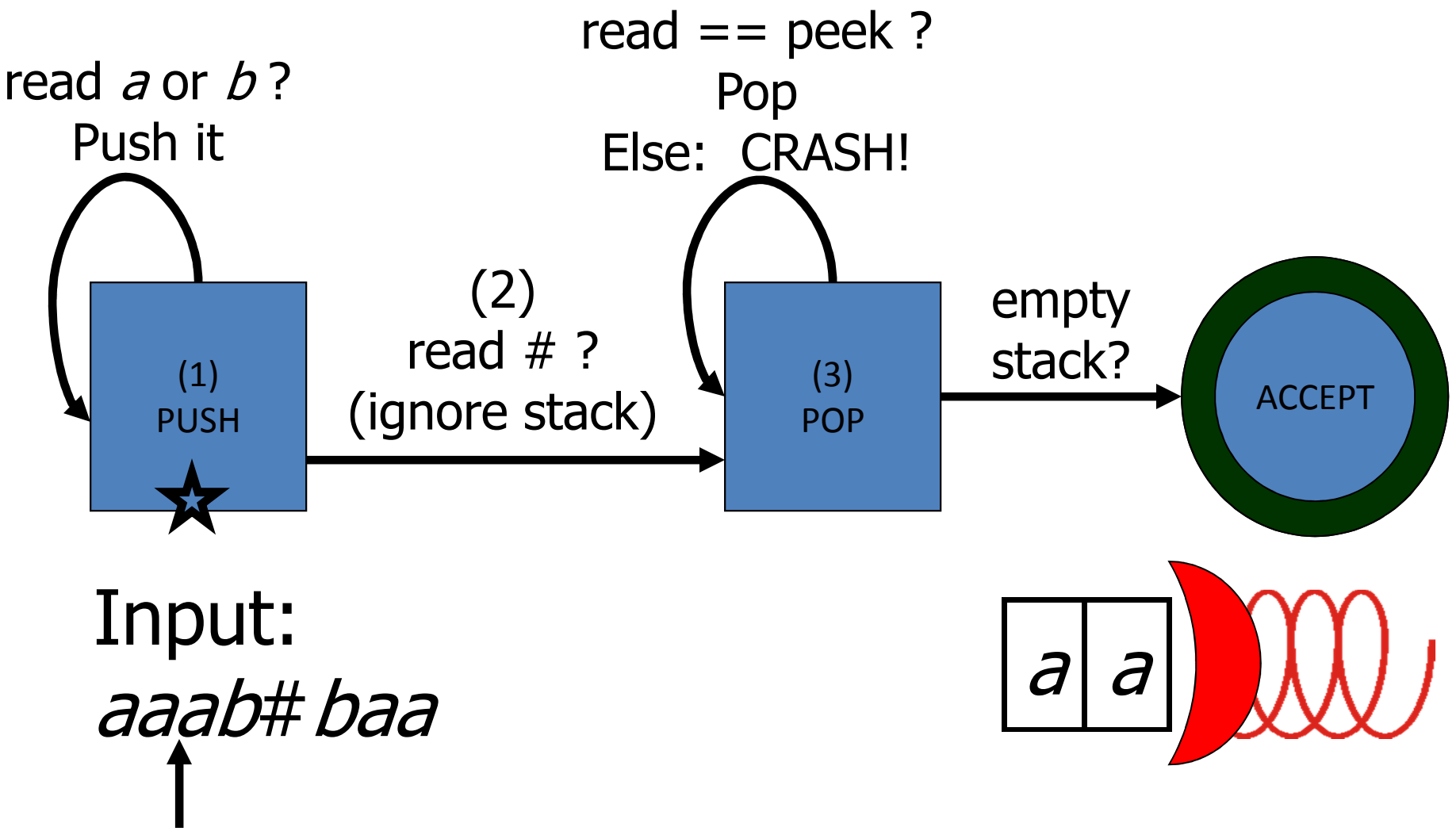

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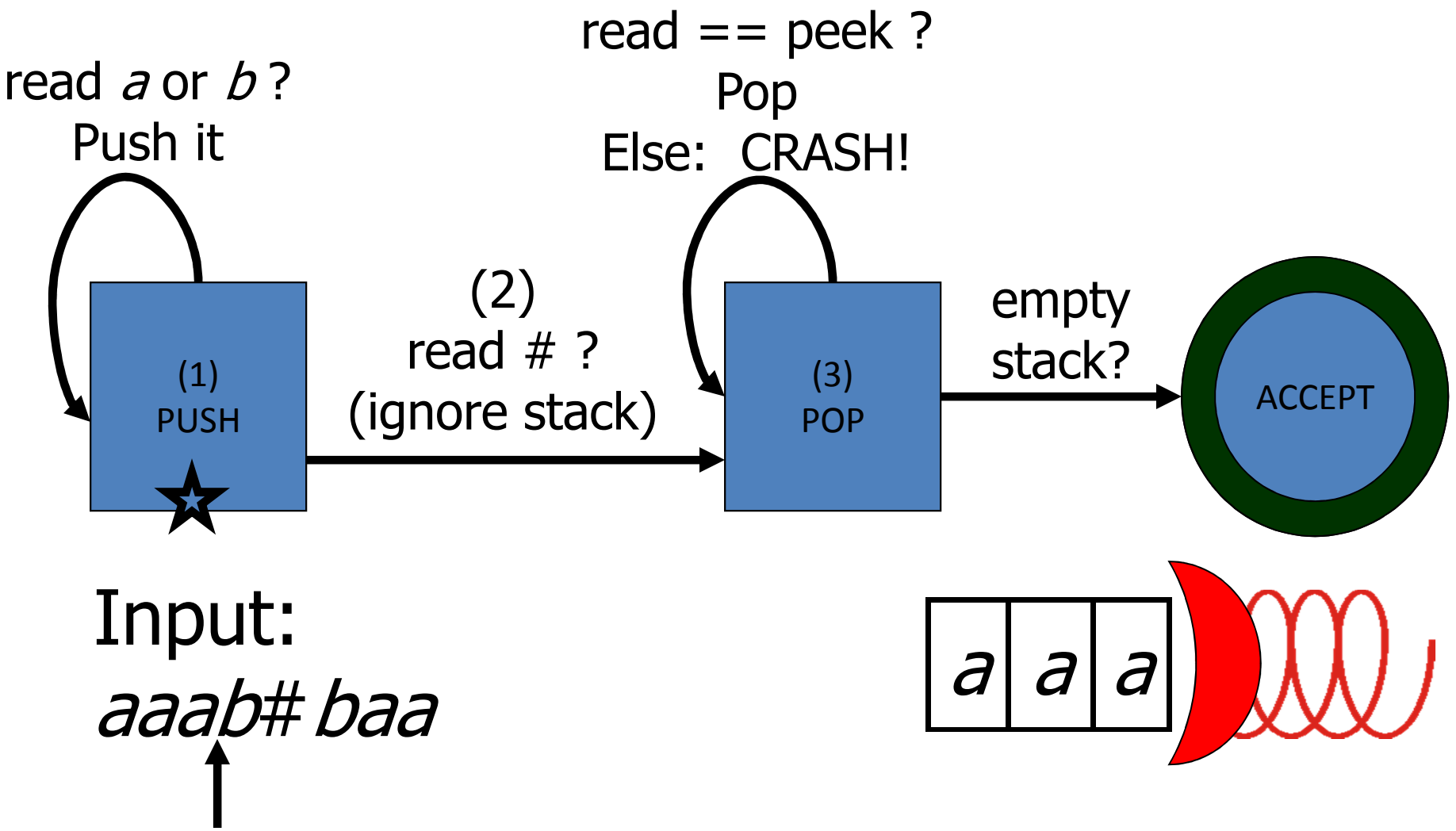

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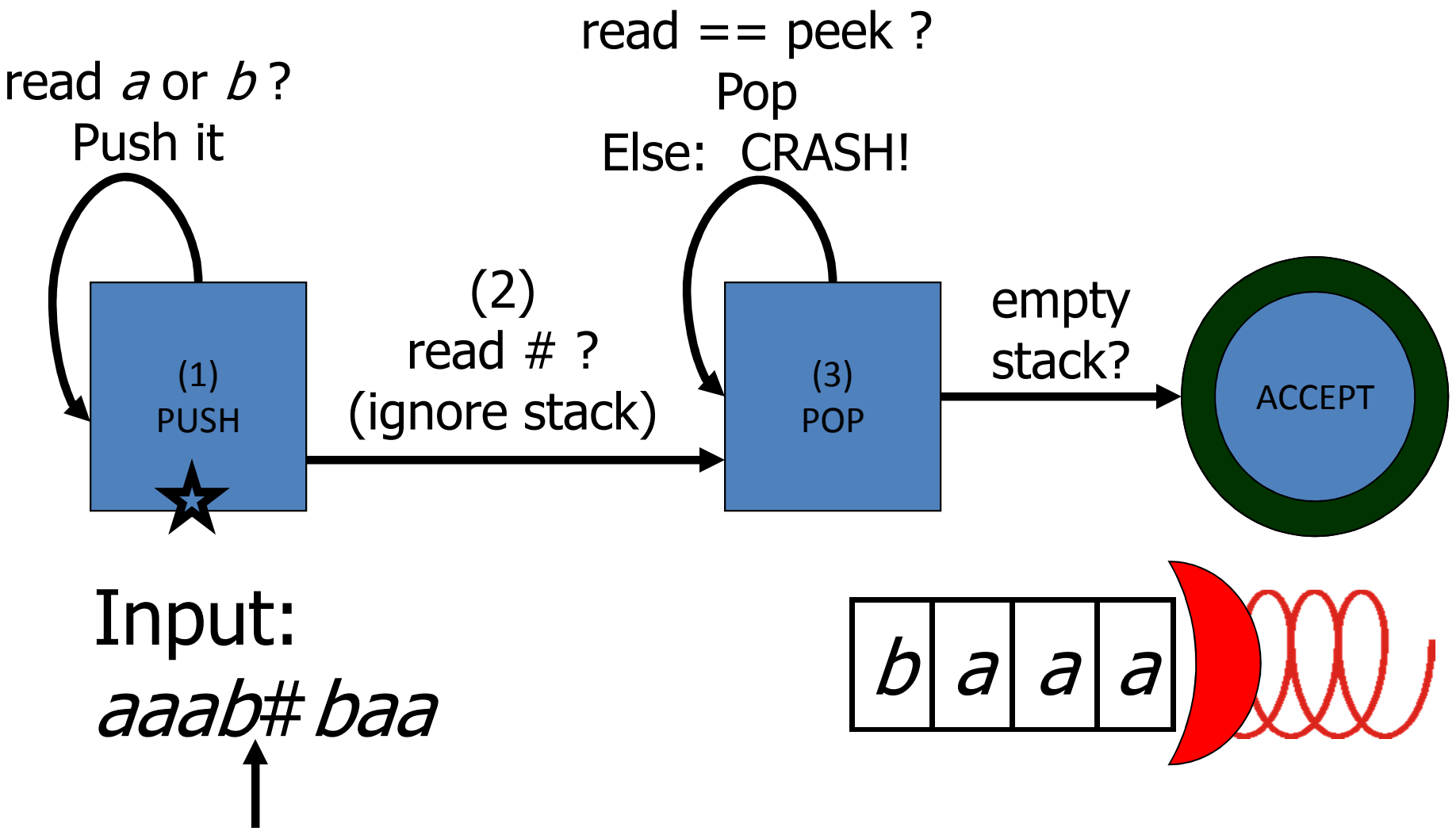

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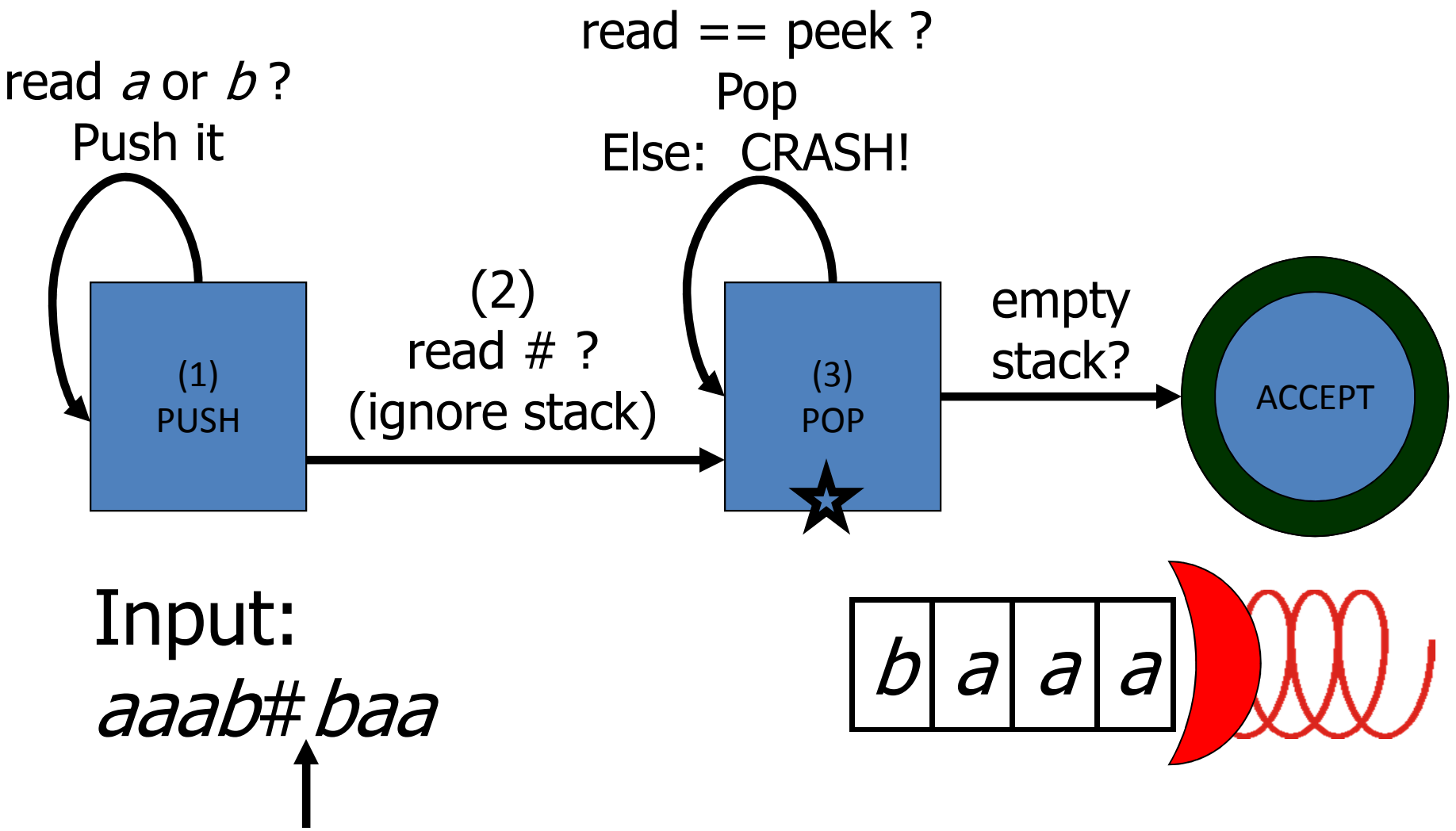

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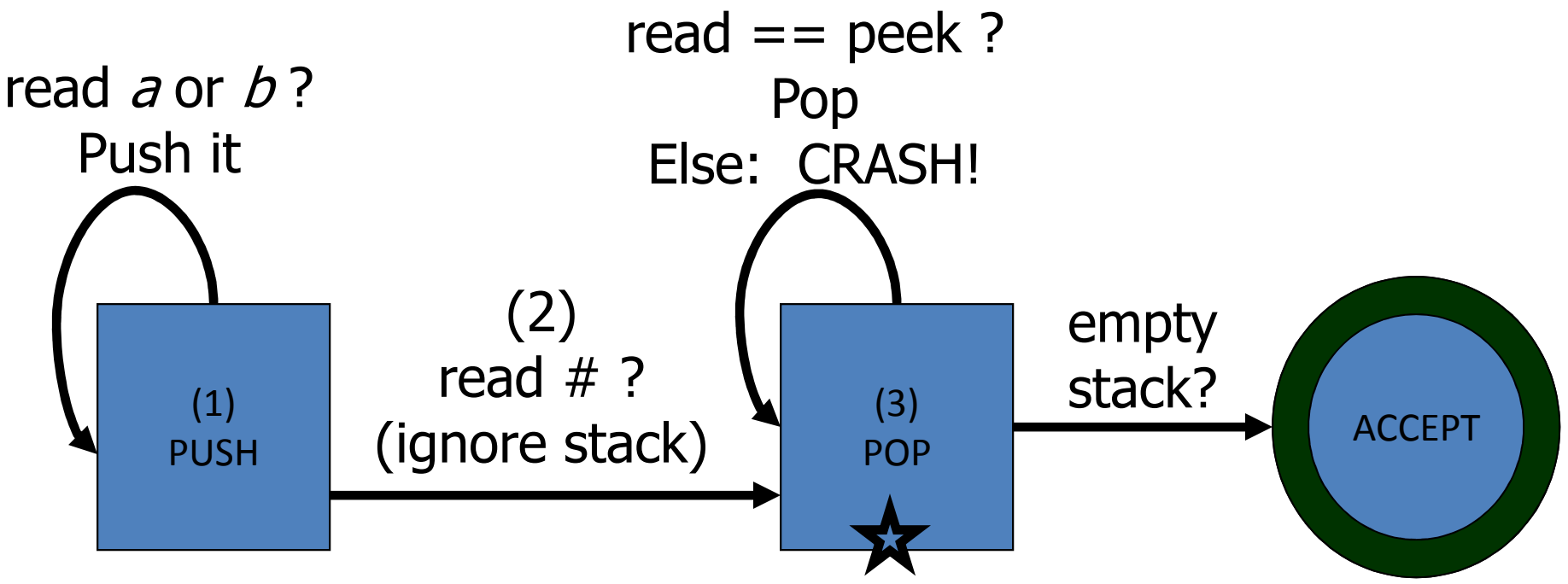
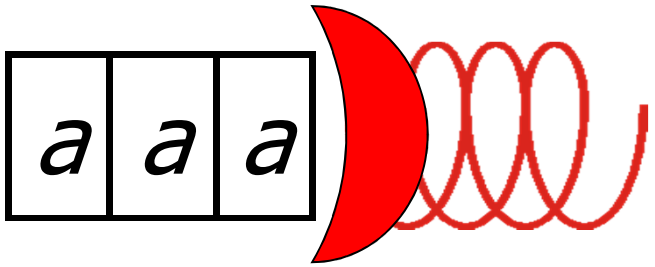


Input:

aaab# baa



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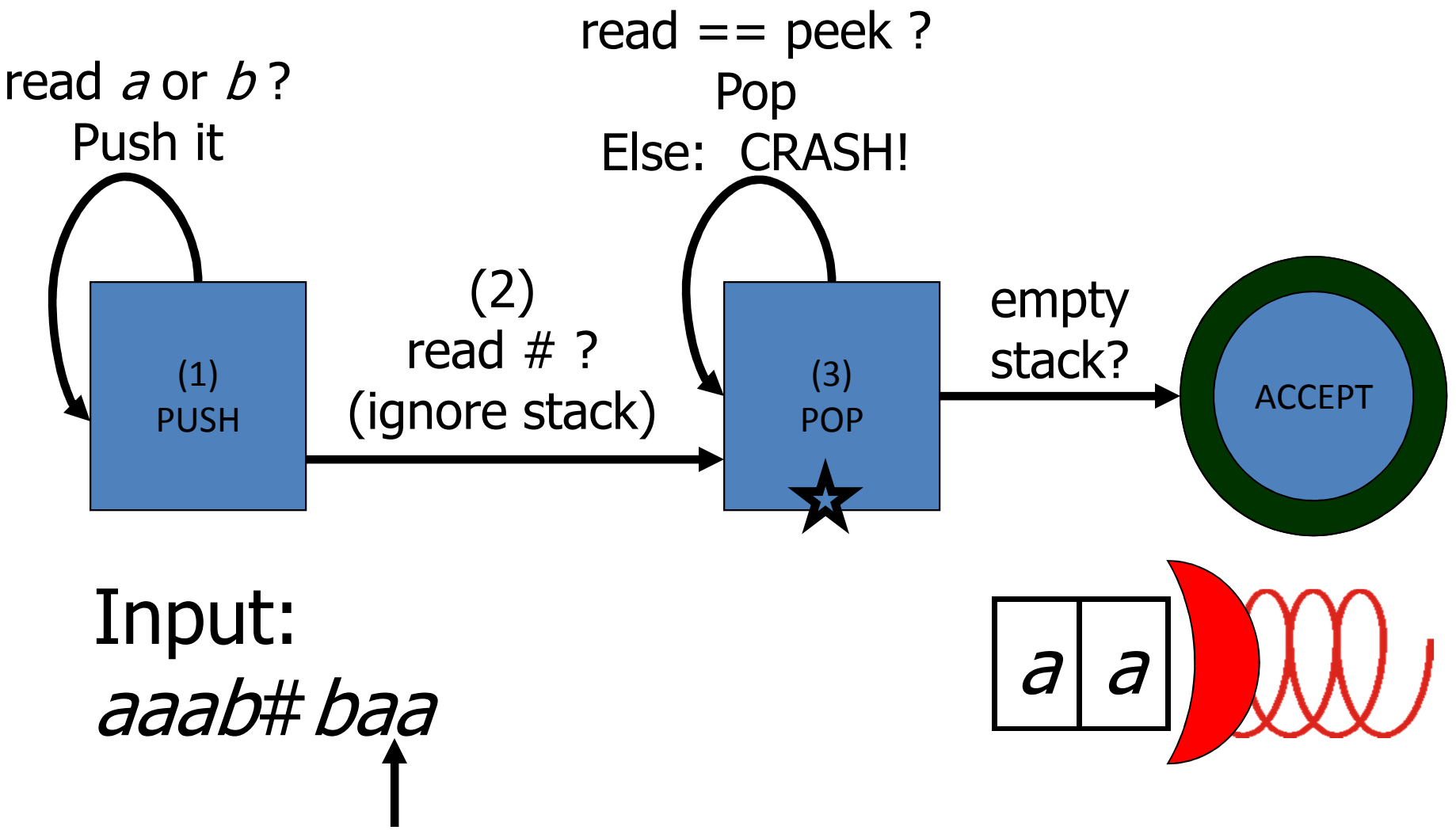

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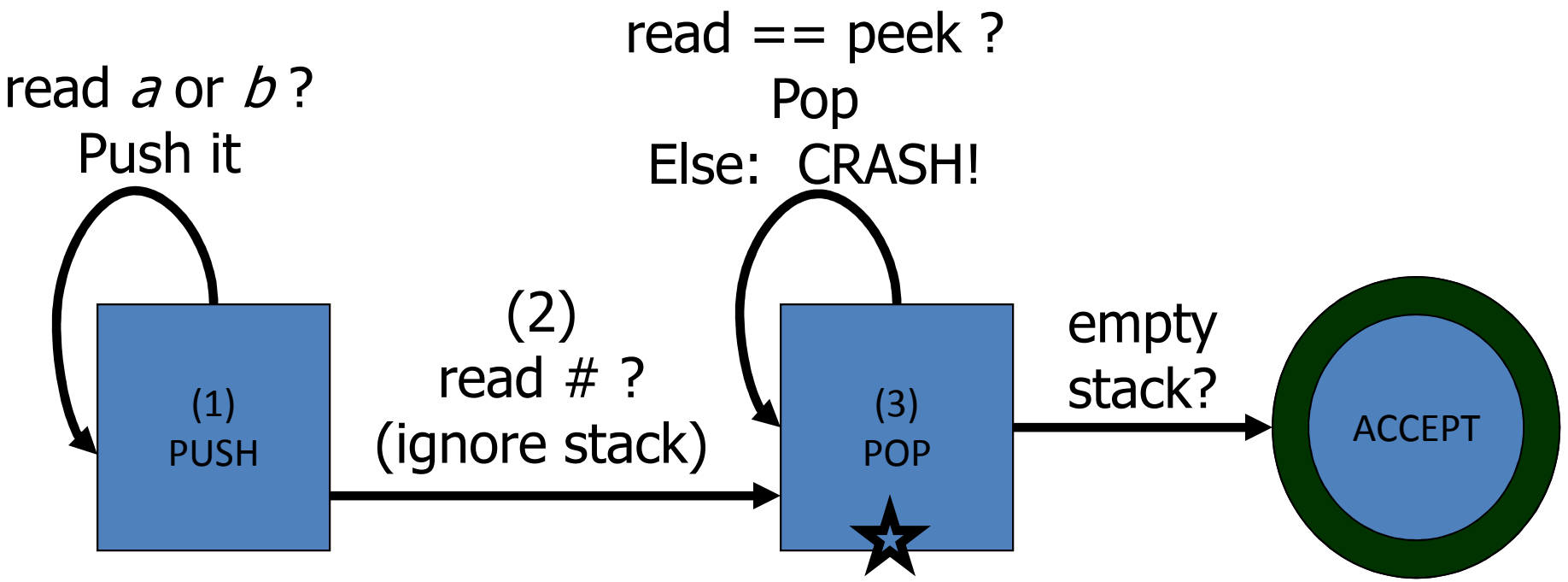
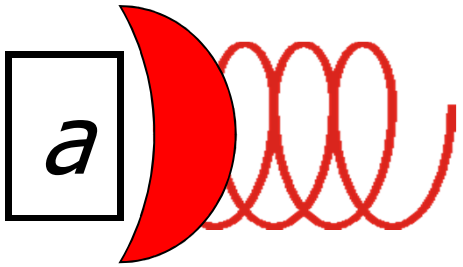


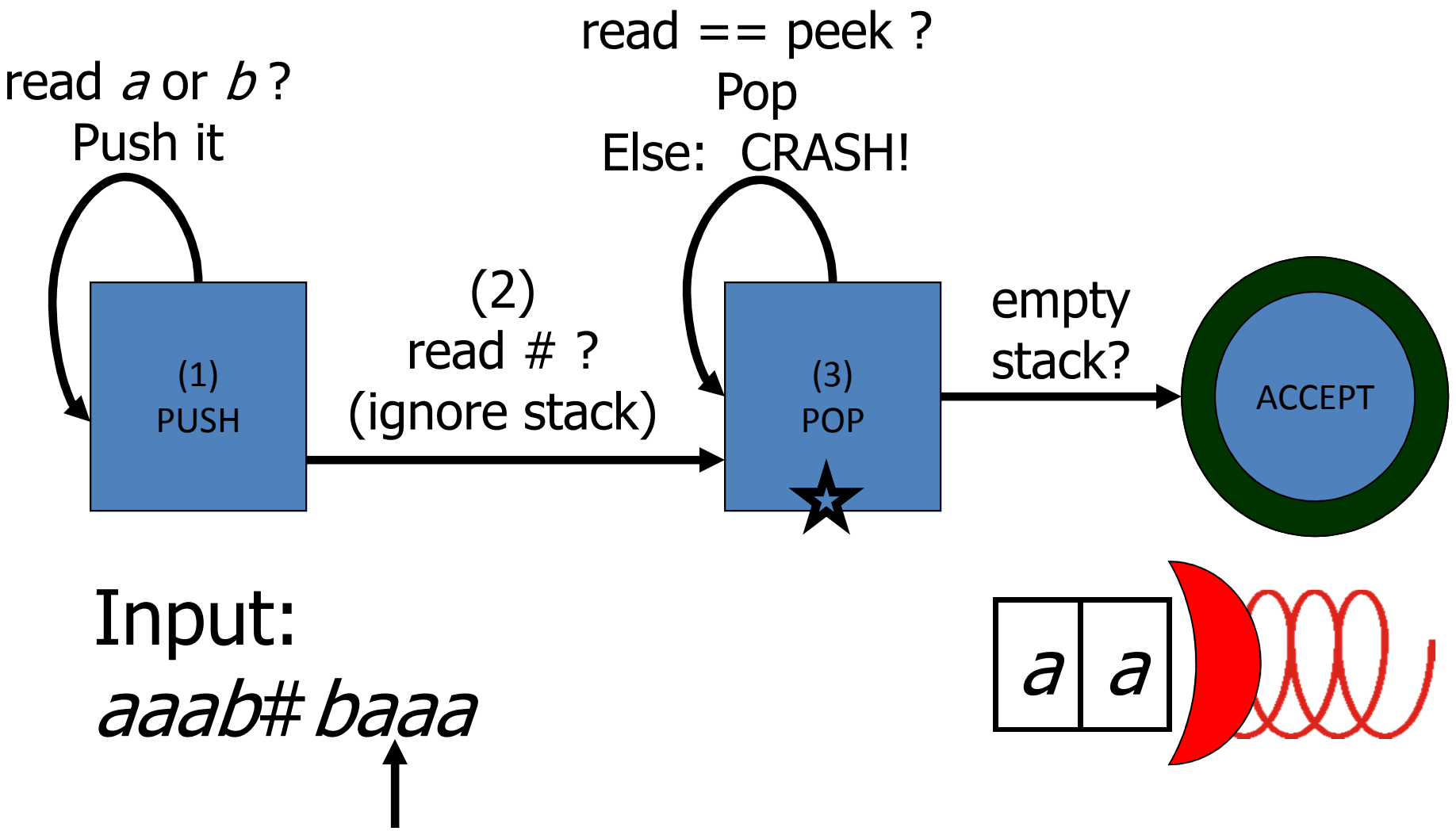
Input:

aaab# baa



REJECT (nonempty stack)

De CFG's para Máquinas de Pilha



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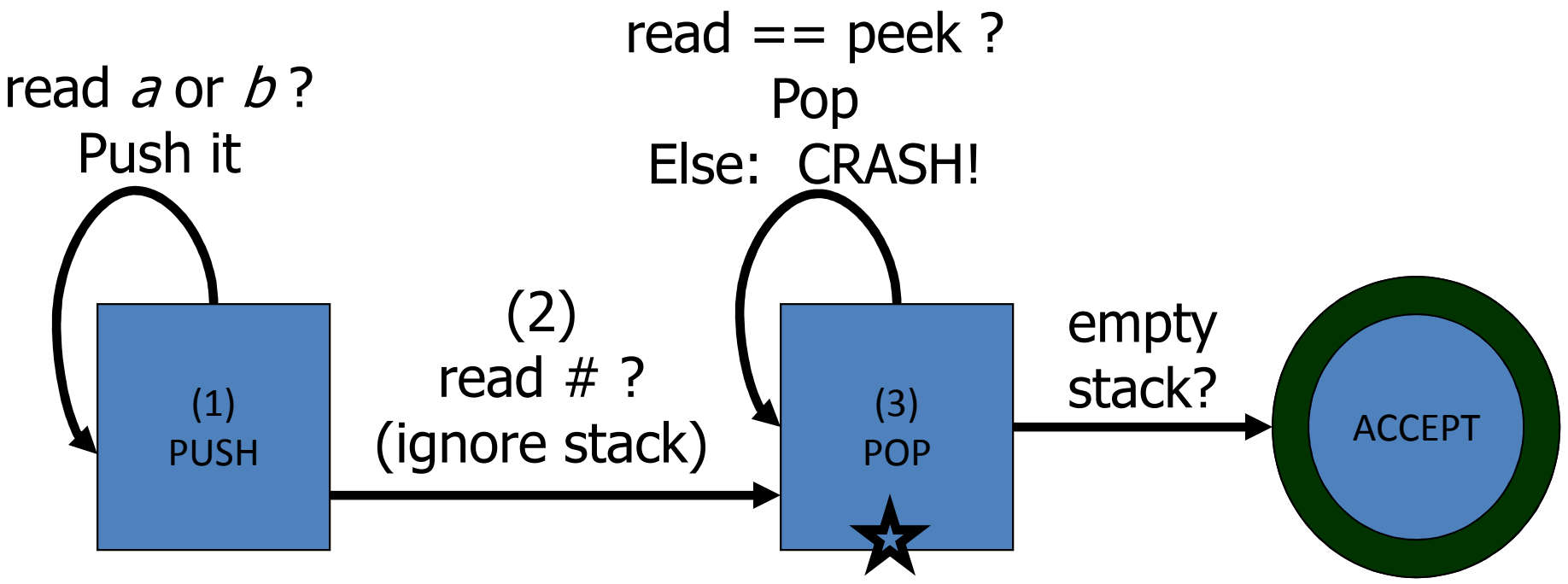
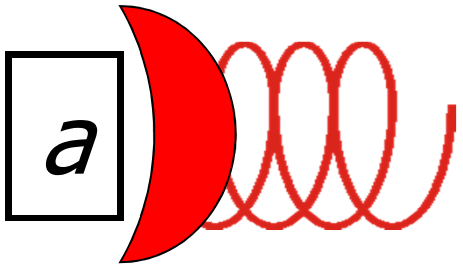


Input:

aaab# baaa



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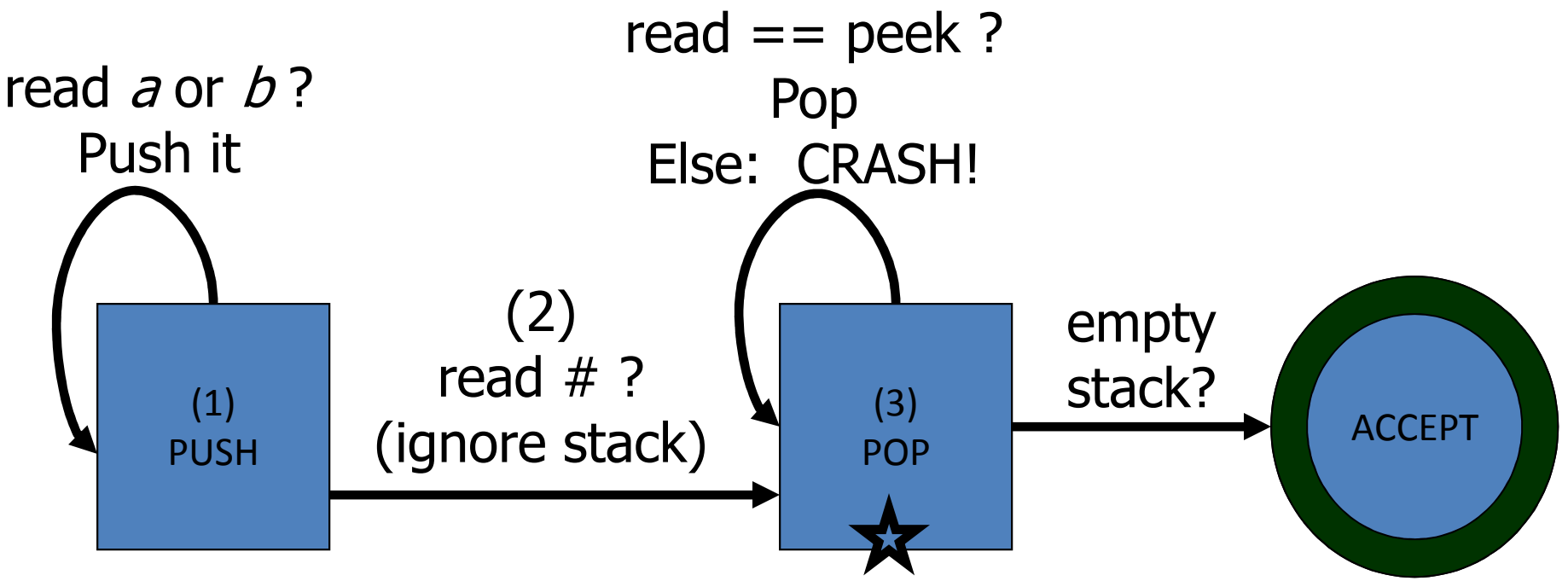
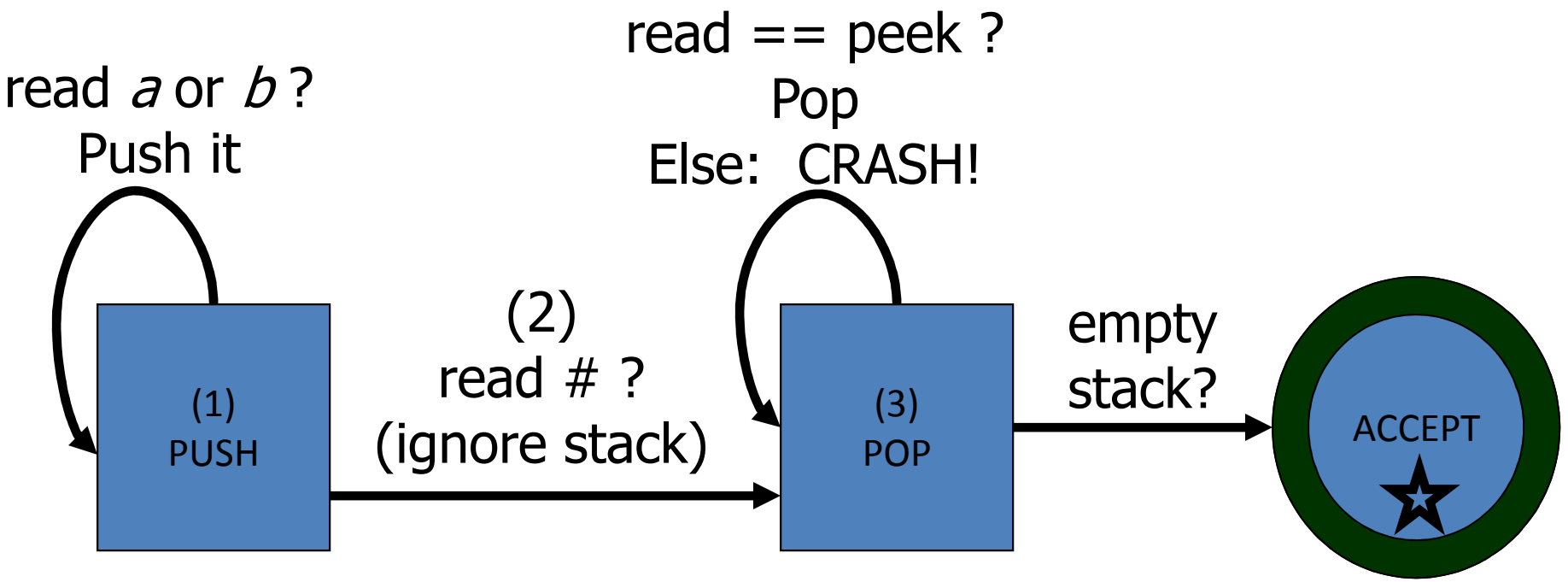


Input:

aaab# baaa

Pause input

De CFG's para Máquinas de Pilha

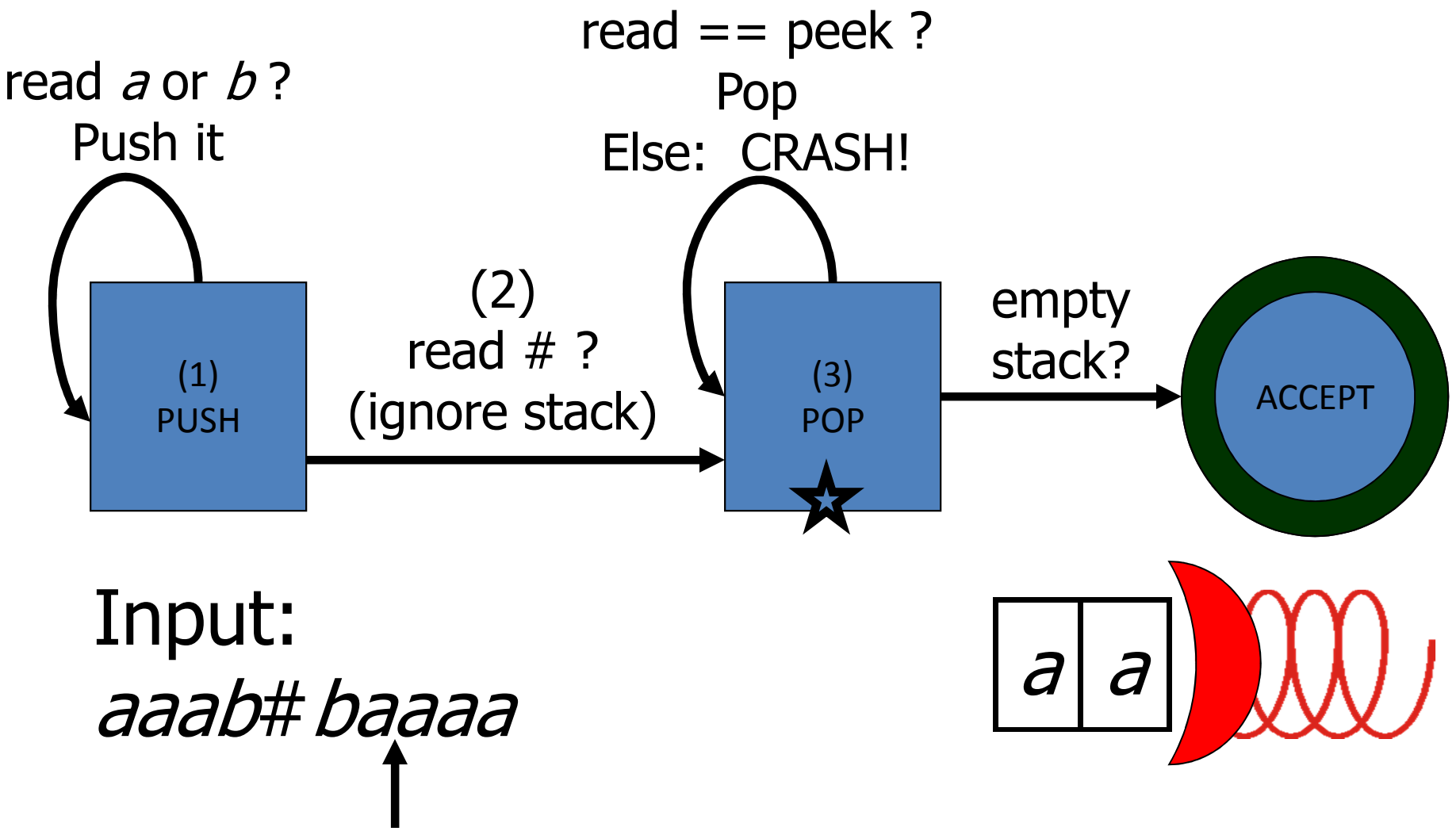


Input:

aaab# baaa

ACCEPT

De CFG's para Máquinas de Pilha



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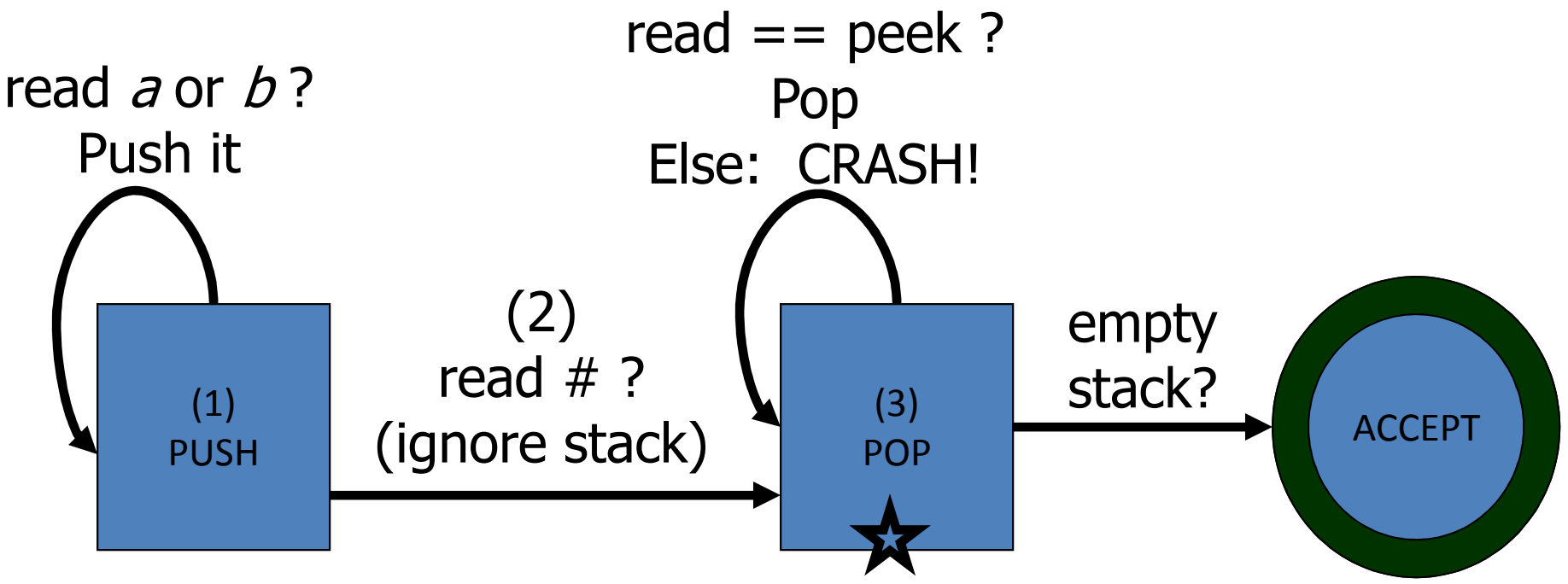
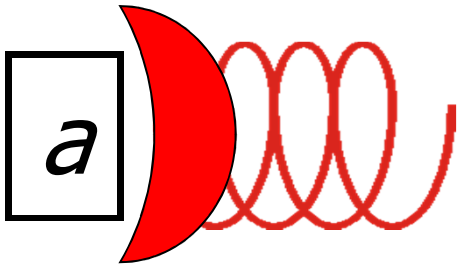


Input:

aaab# baaaa



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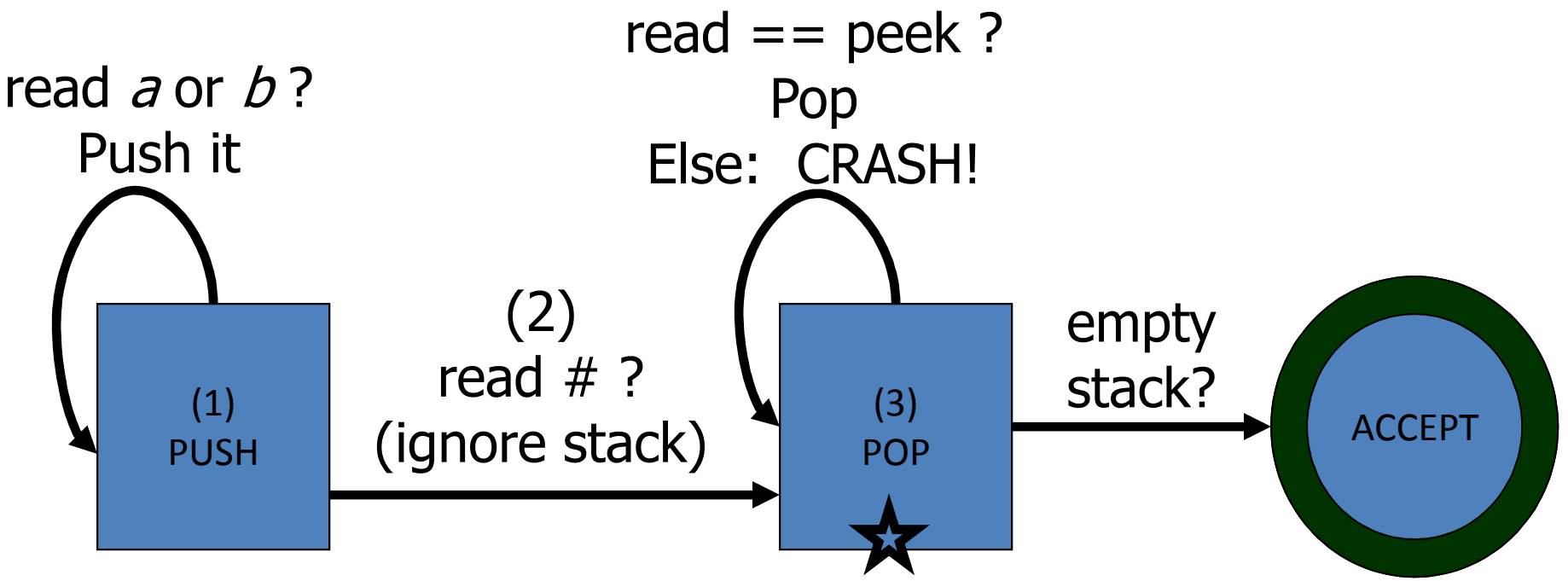
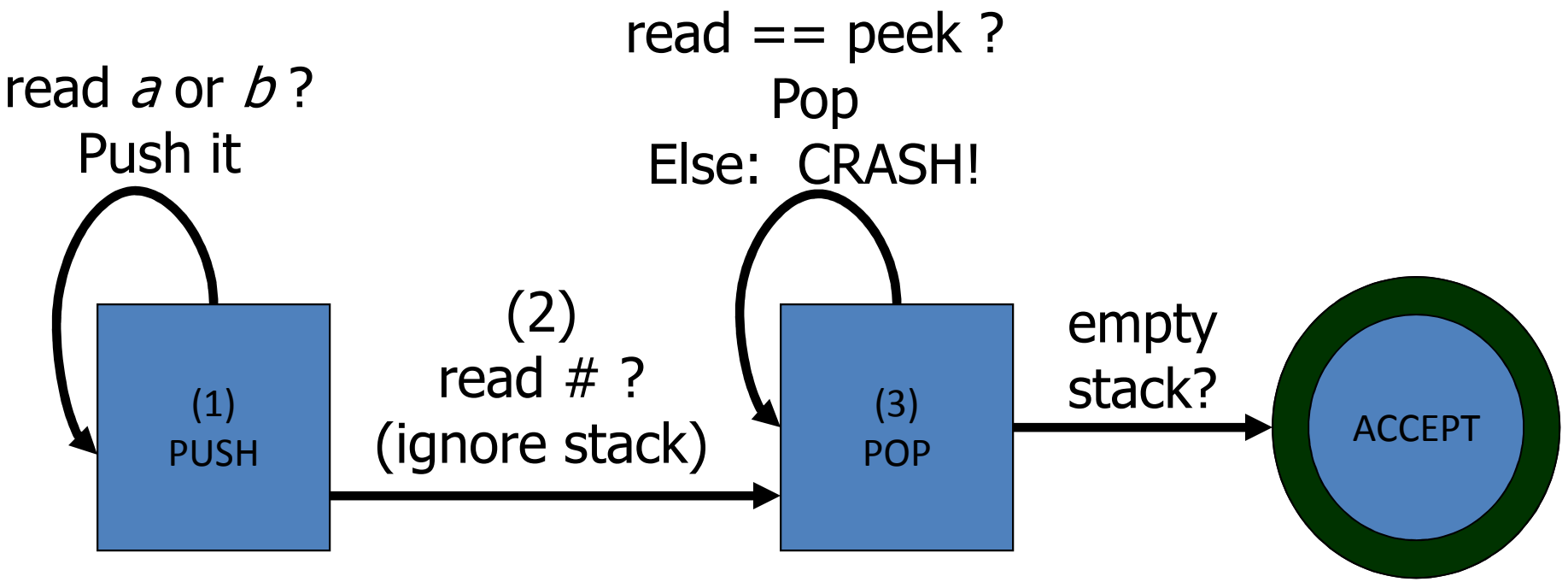


Input:

aaab# baaaa

Pause input

De CFG's para Máquinas de Pilha



Input:

aaab# baaaa

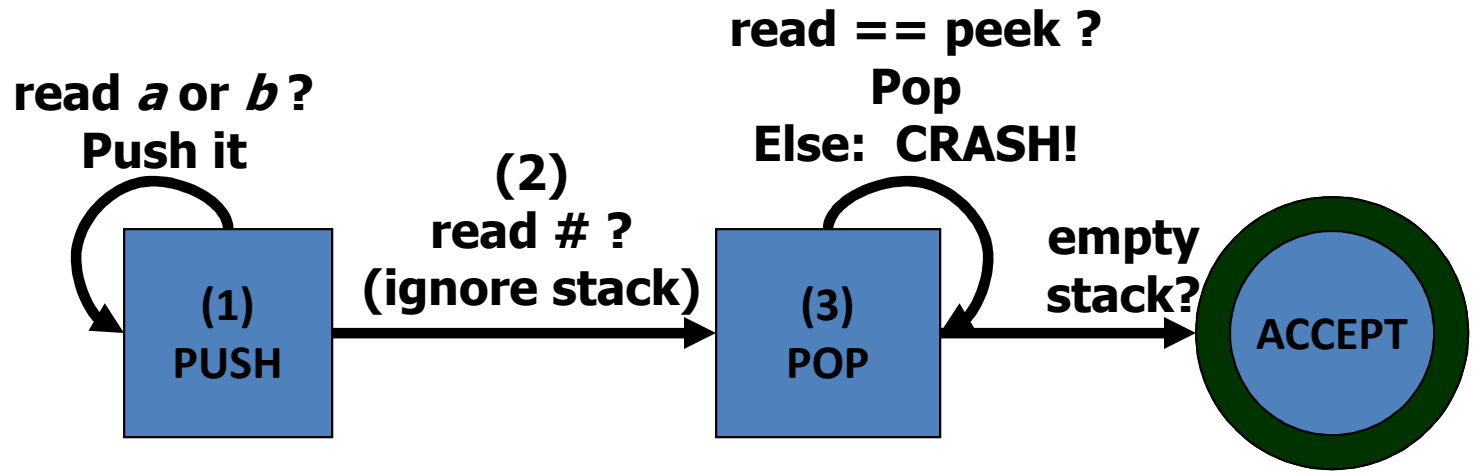
CRASH

PDA's à la Sipser

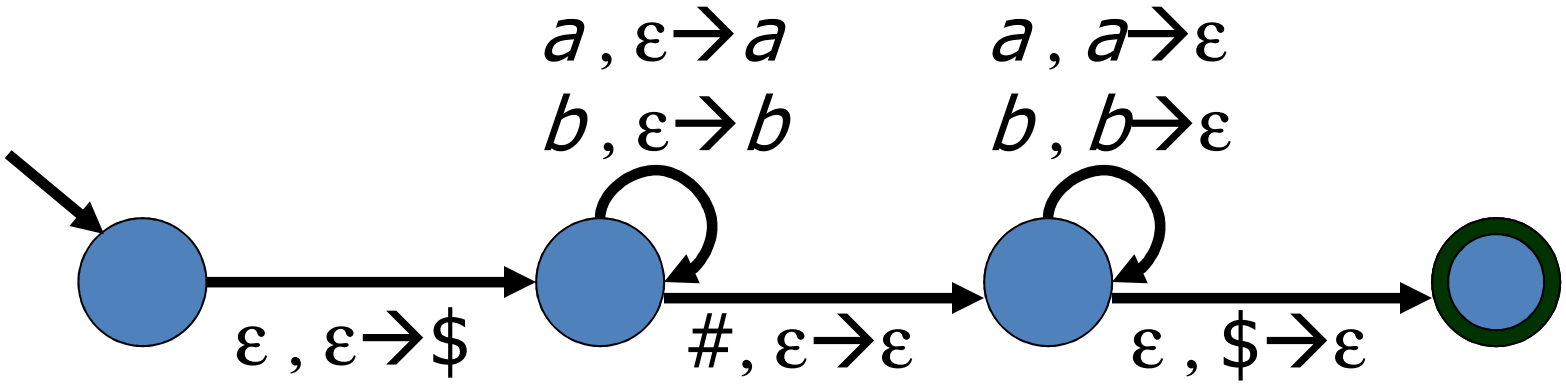
Para facilitar a análise, máquinas de pilha teóricas têm um conjunto restrito de operações. Cada livro-author tem sua própria versão. PDAs descritos em Sipser são assim:

- Push/Pop agrupados em única operação: *substituir o símbolo no topo da pilha*
- Nenhum teste intrínsico de pilha vazia
- Epsilon é usado para aumentar a funcionalidade: máquinas *não deterministas.*

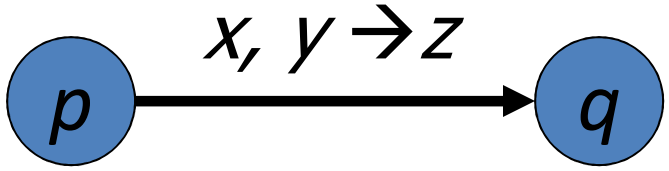
Versão Sipser's



Torna-se:



Versão Sipser's



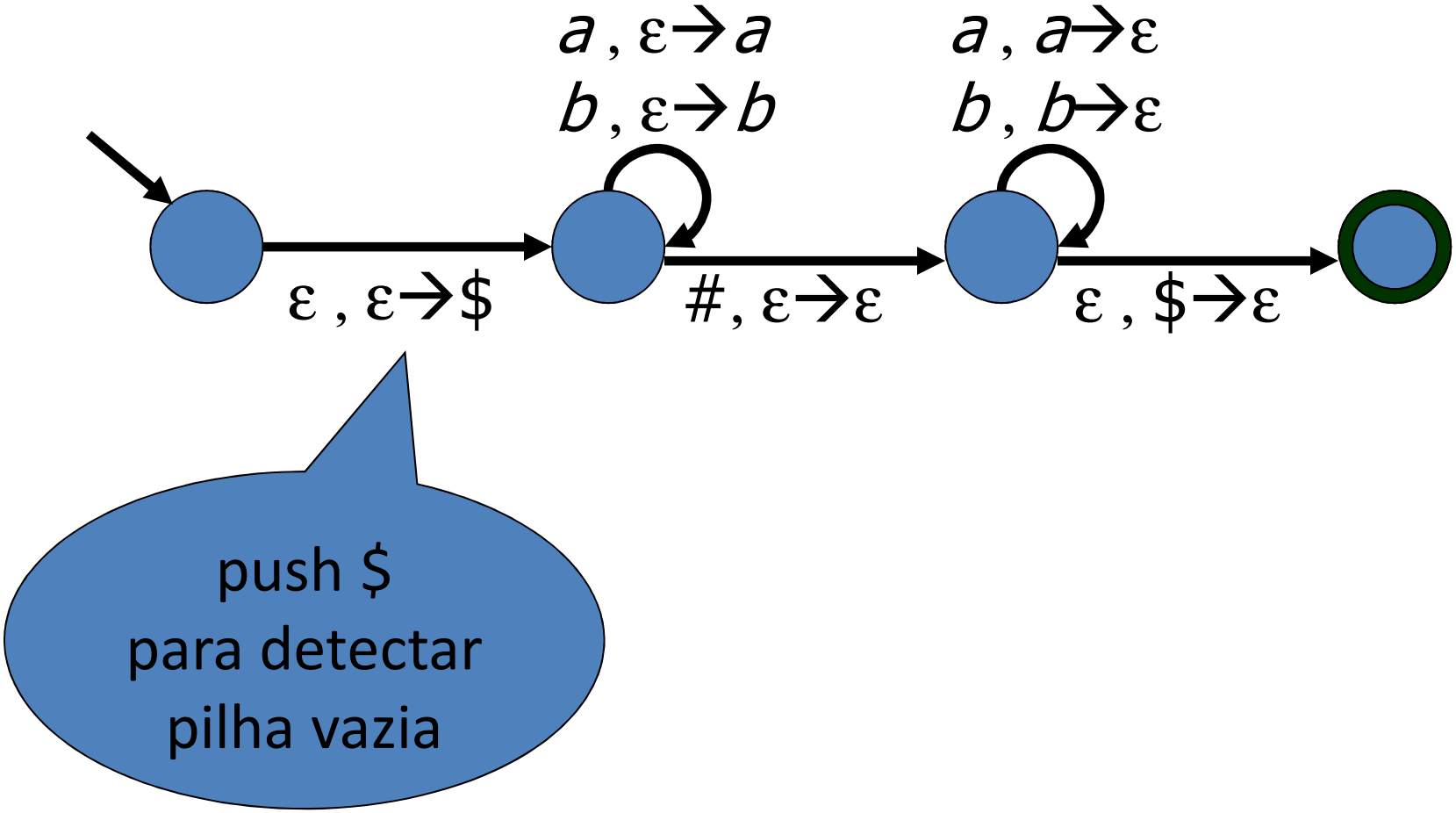
Significado da convenção do rótulo:

Se está no estado *p* **e** o próximo símbolo é *x* **e** o topo da pilha é *y*,

então vá para *q* **e** substitua *y* por *z* na pilha.

- *x* = ε: ignore a entrada, não leia
- *y* = ε: ignore o topo da pilha e empilhe *z*
- *z* = ε: desempilhe *y*

Versão Sipser's



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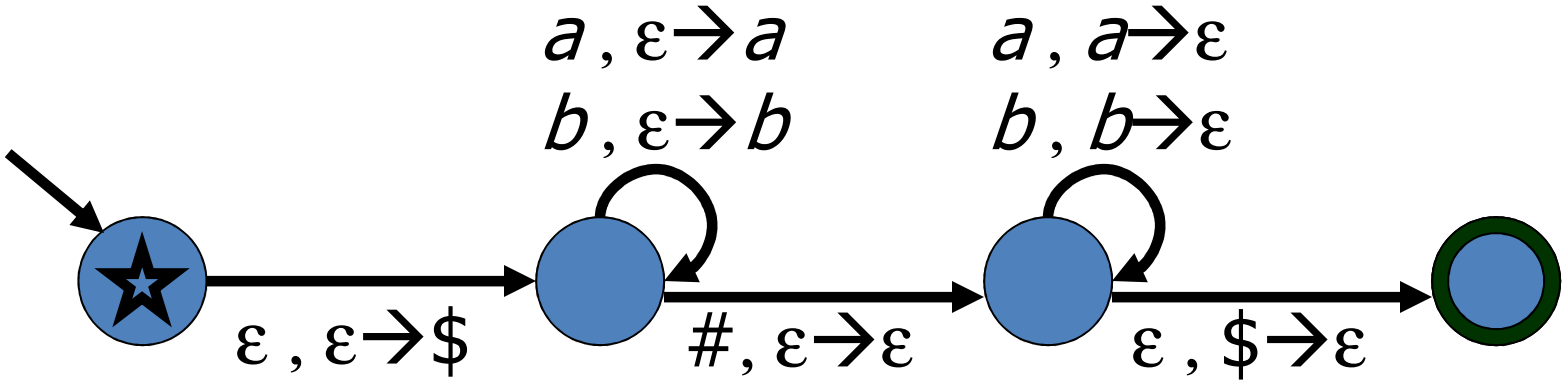
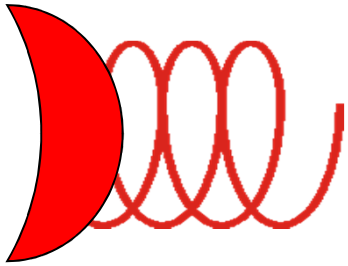


Input:

aaab# baaa



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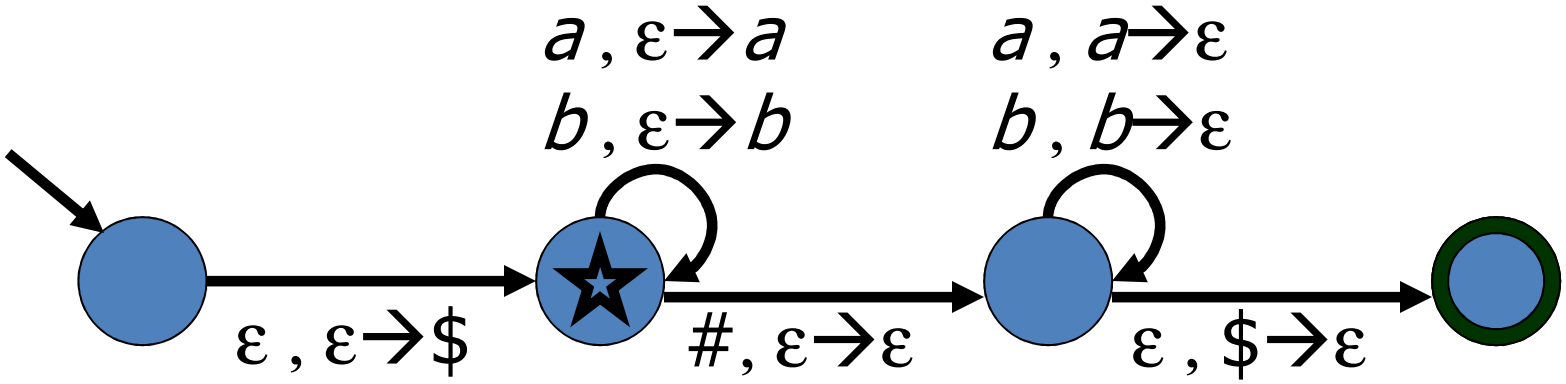
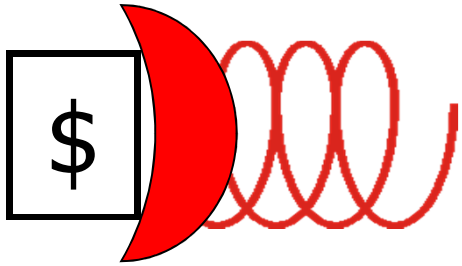


Input:

aaab# baaa



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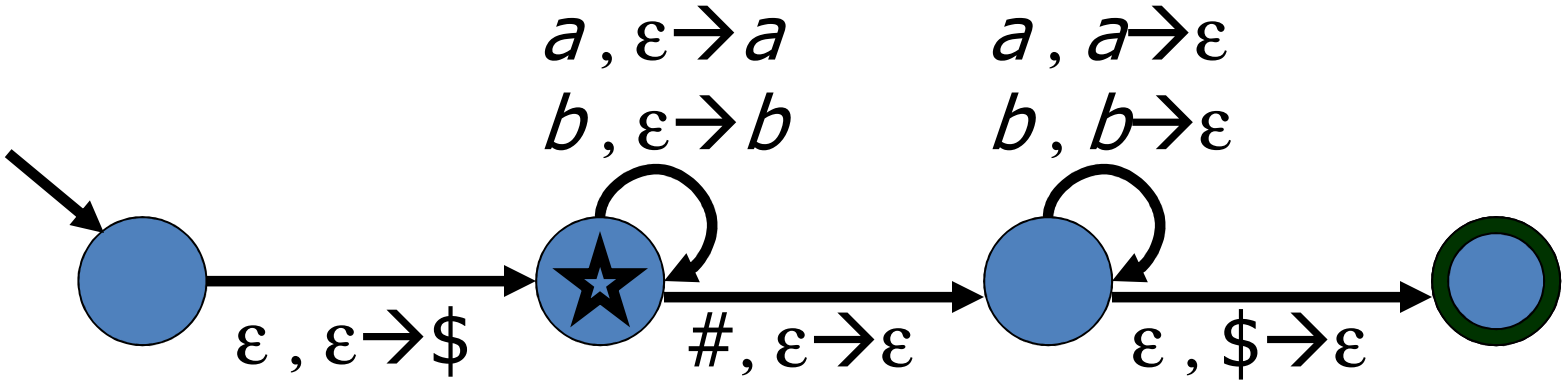

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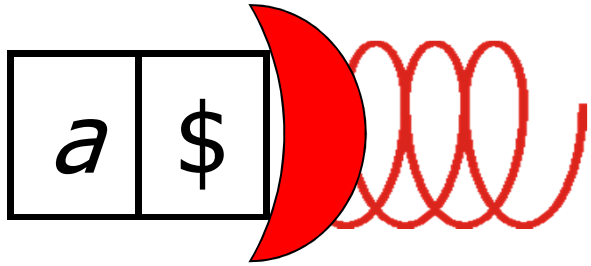
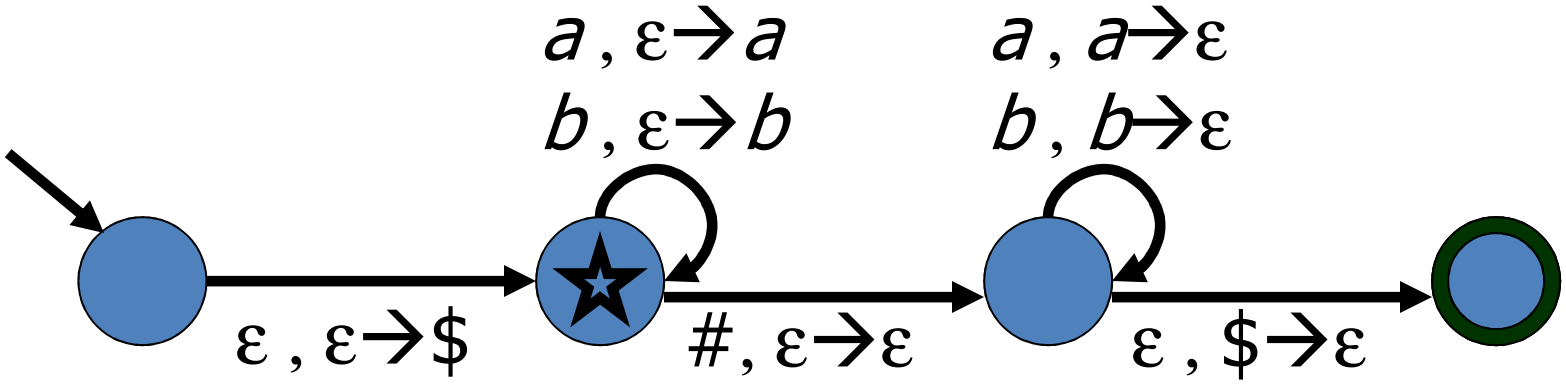


Versão Sipser's



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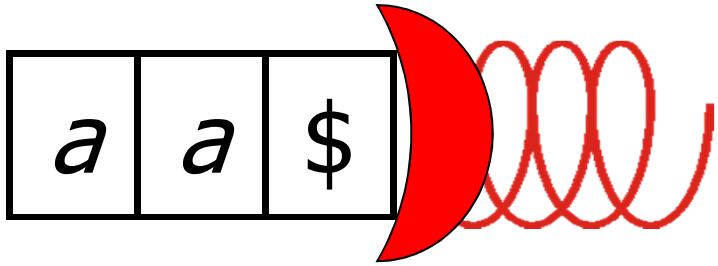

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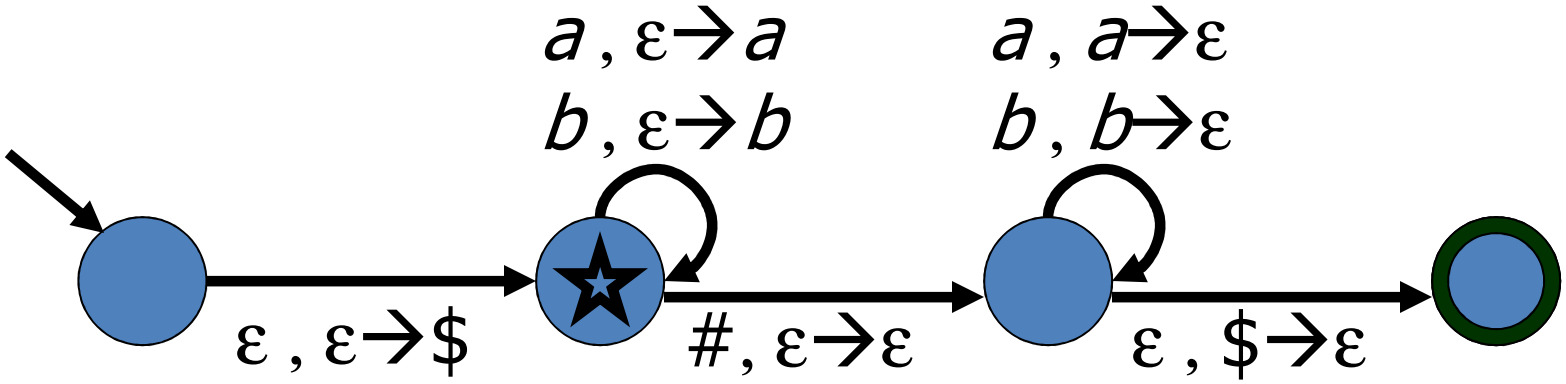

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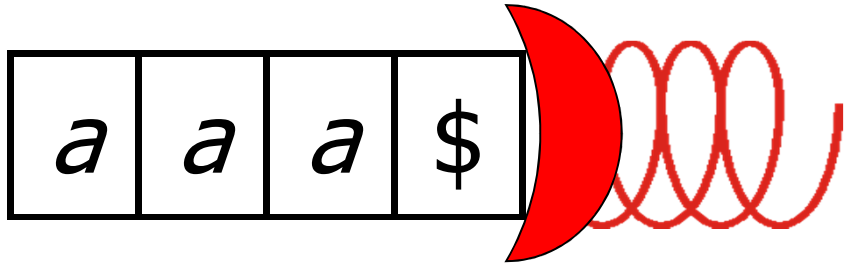

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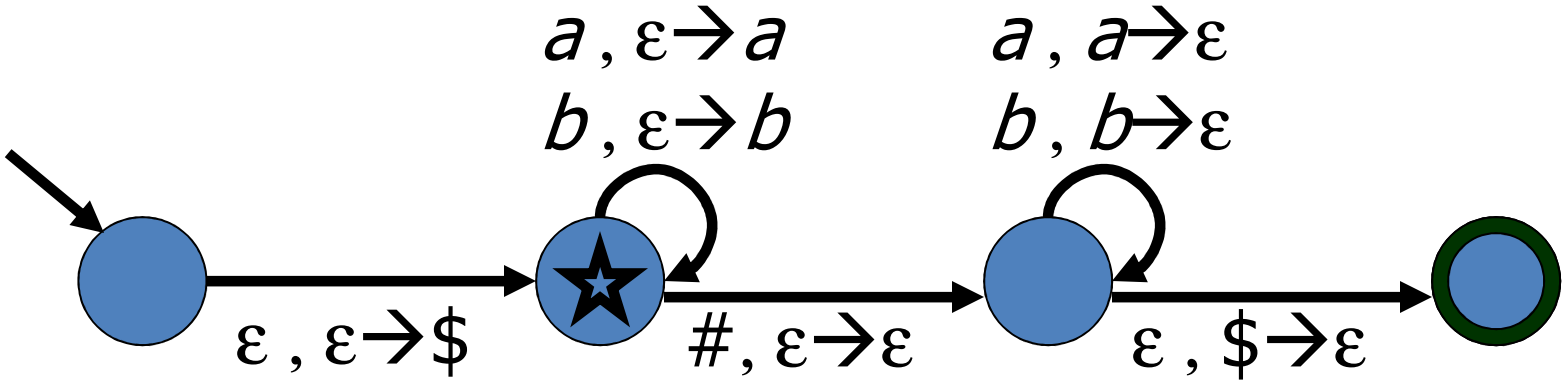

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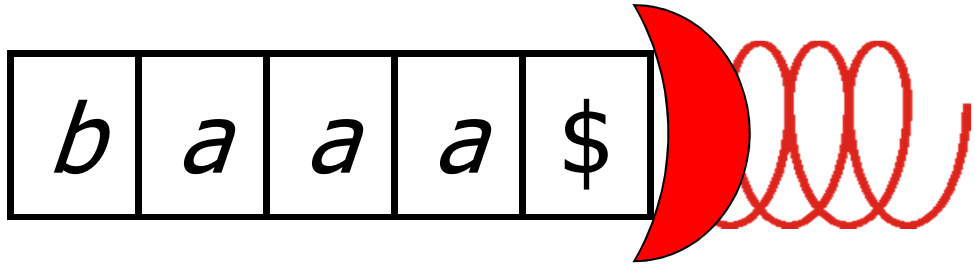

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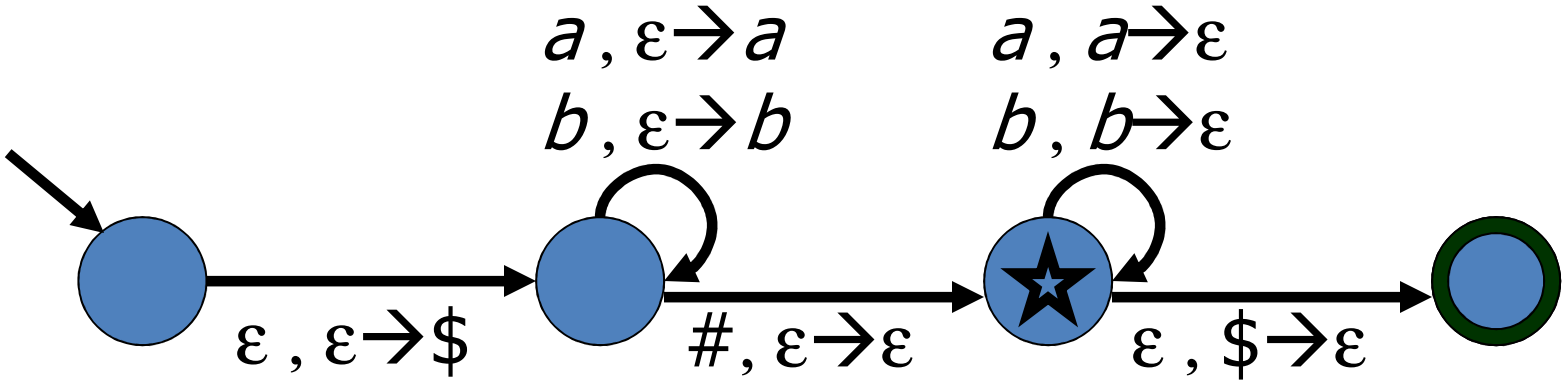

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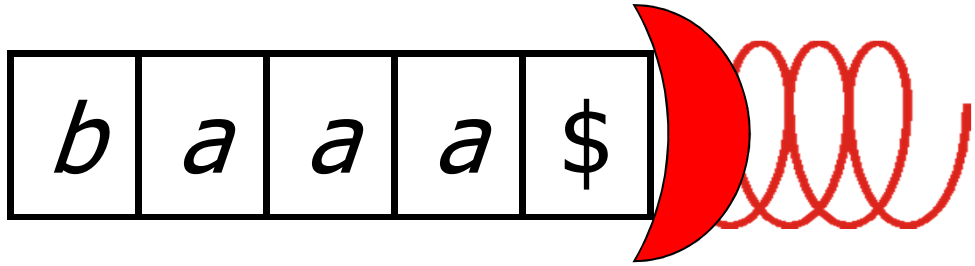

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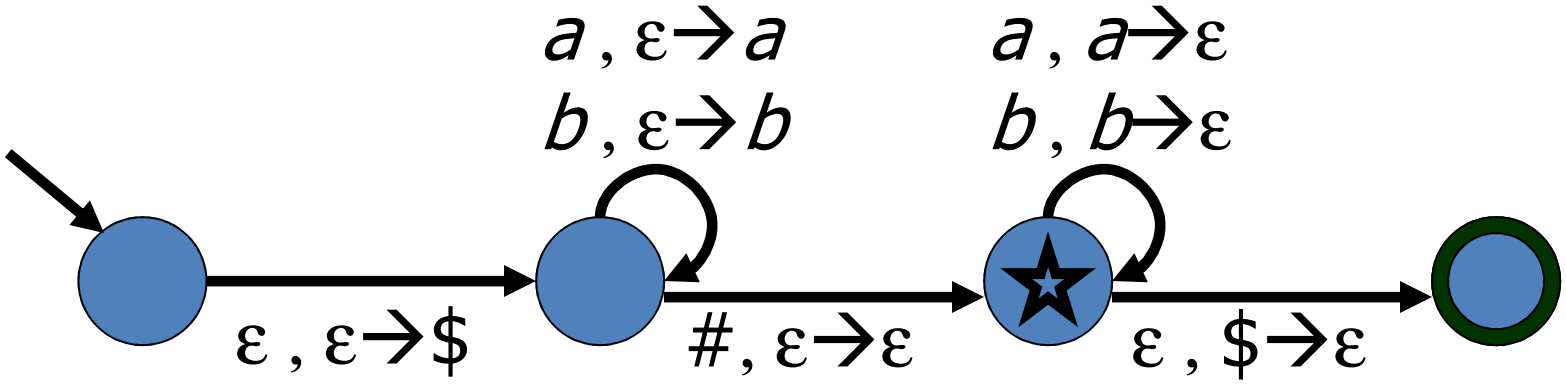

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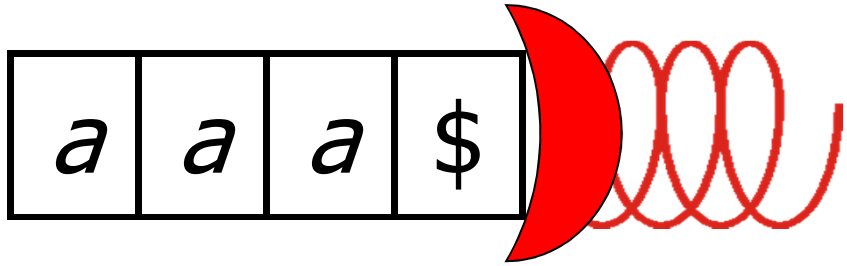

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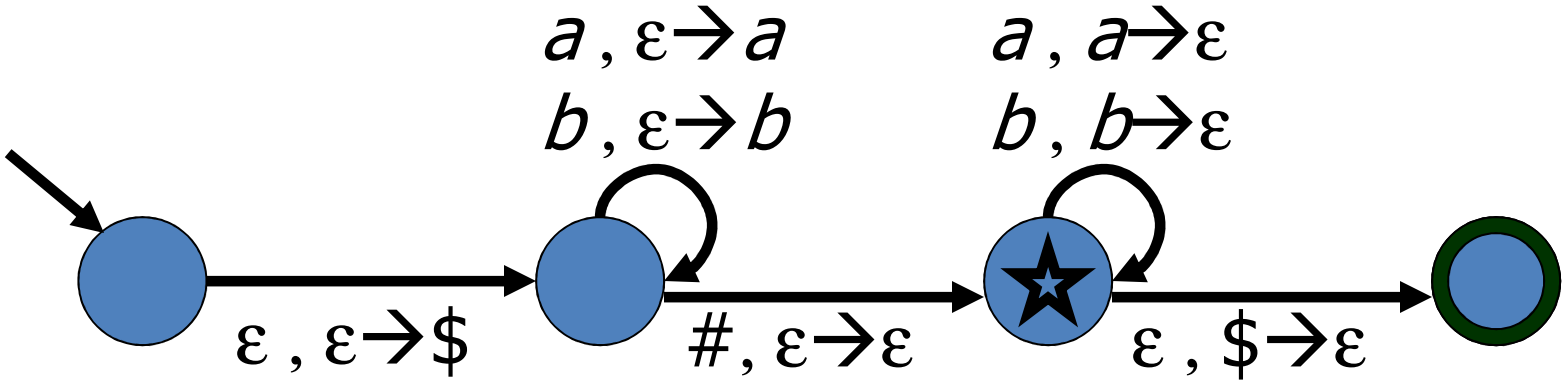

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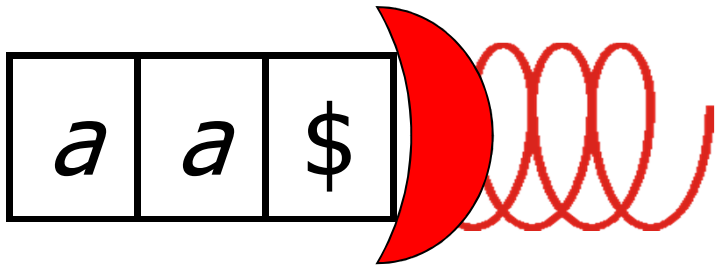

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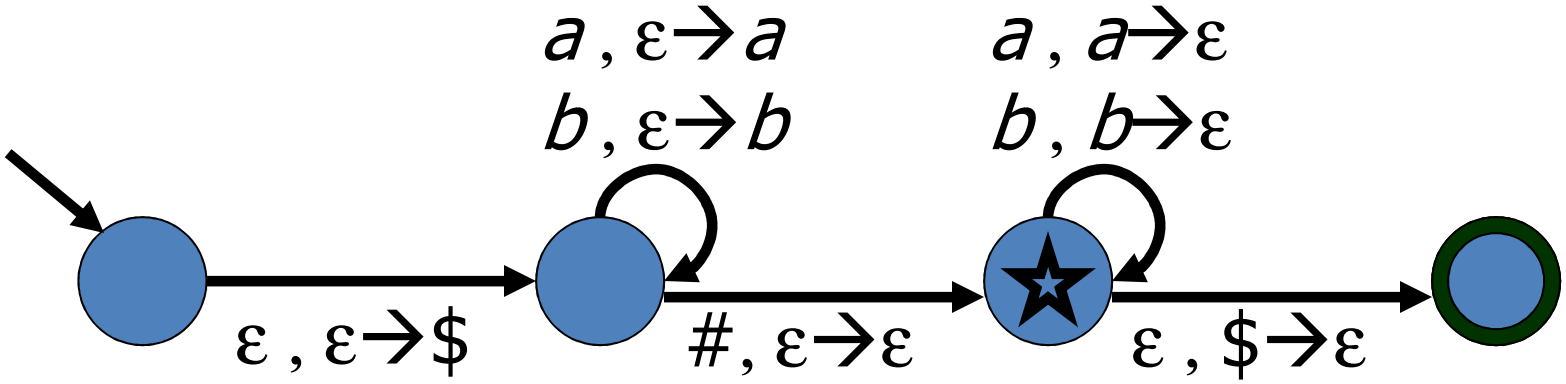

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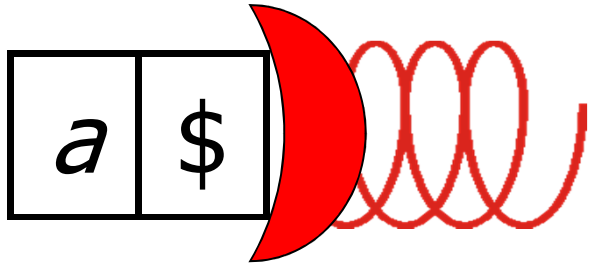

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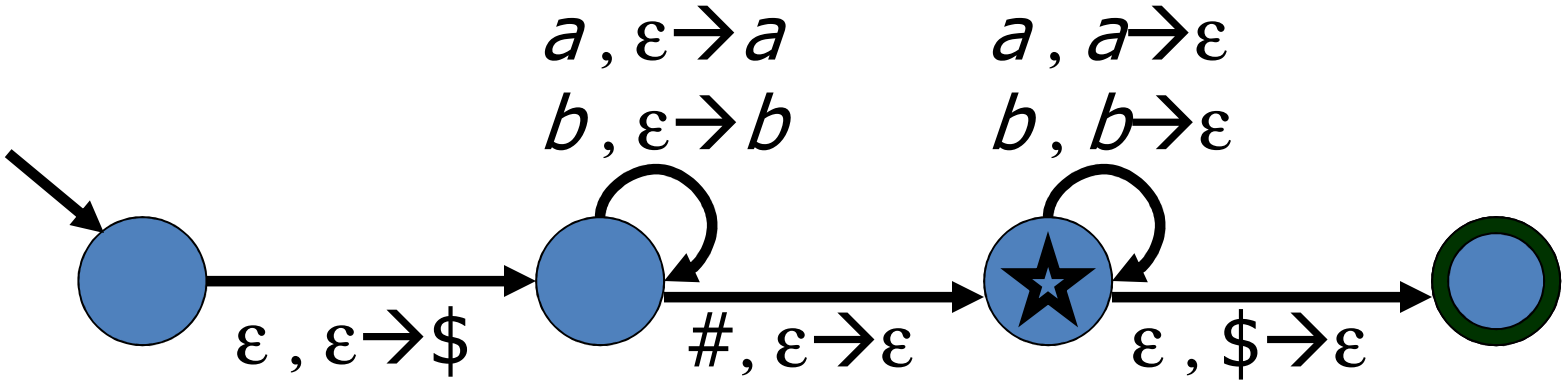

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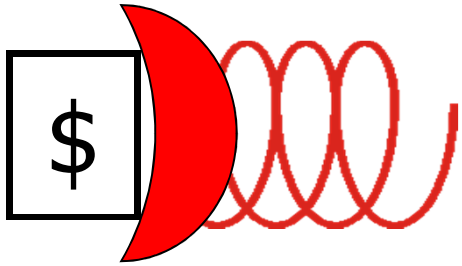

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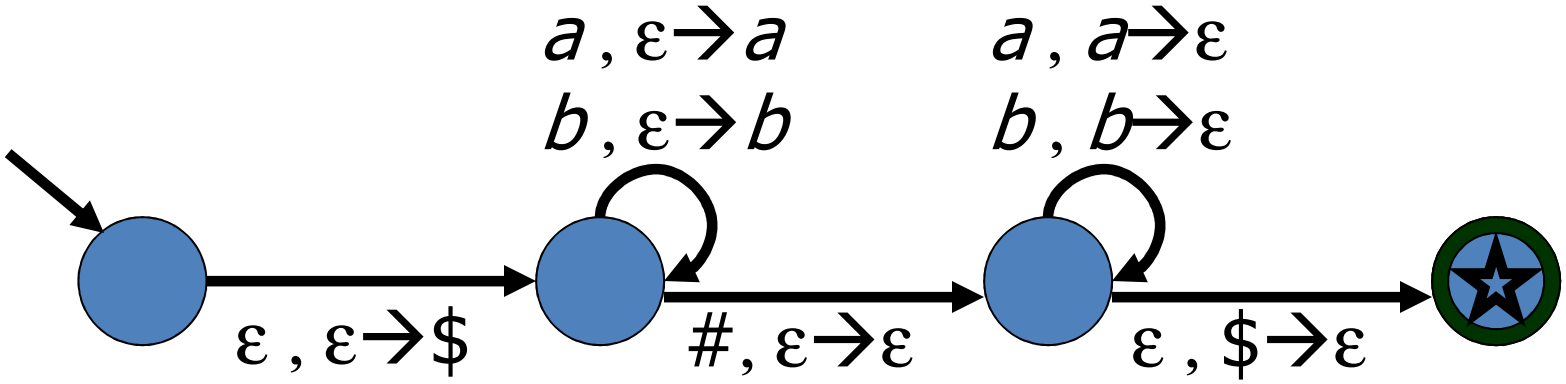

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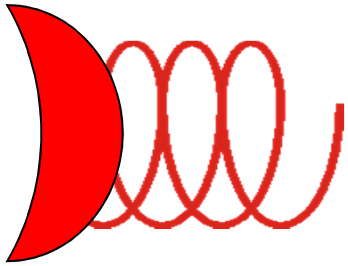
ACCEPT!



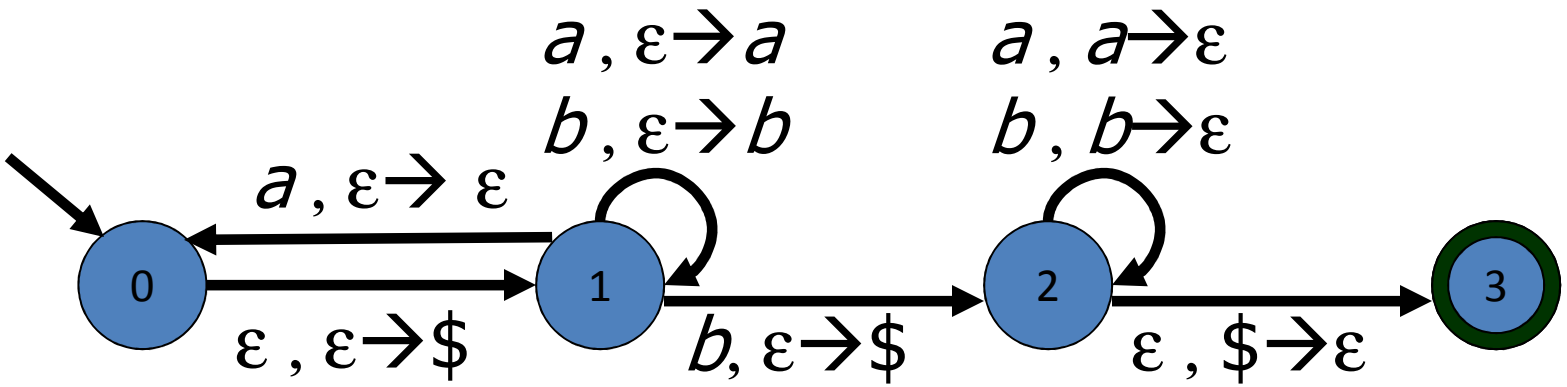
PDA
Definição Formal

DEF: Um ***autômato de pilha*** (PDA) é uma 6-tupla $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ onde Q, Σ e q_0, são como para um AF. Γ é o ***alpfabeto da pilha***. δ é uma função:

$$\delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \rightarrow P(Q \times \Gamma_\varepsilon)$$

Portanto, dado um estado *p,* uma símbolo lido *x* e um símbolo de pilha *y*, $\delta(p,x,y)$ retorna todo (q,z) tal que *q* é um estado alvo e *z* é um símbolo que deve substituir *y*.

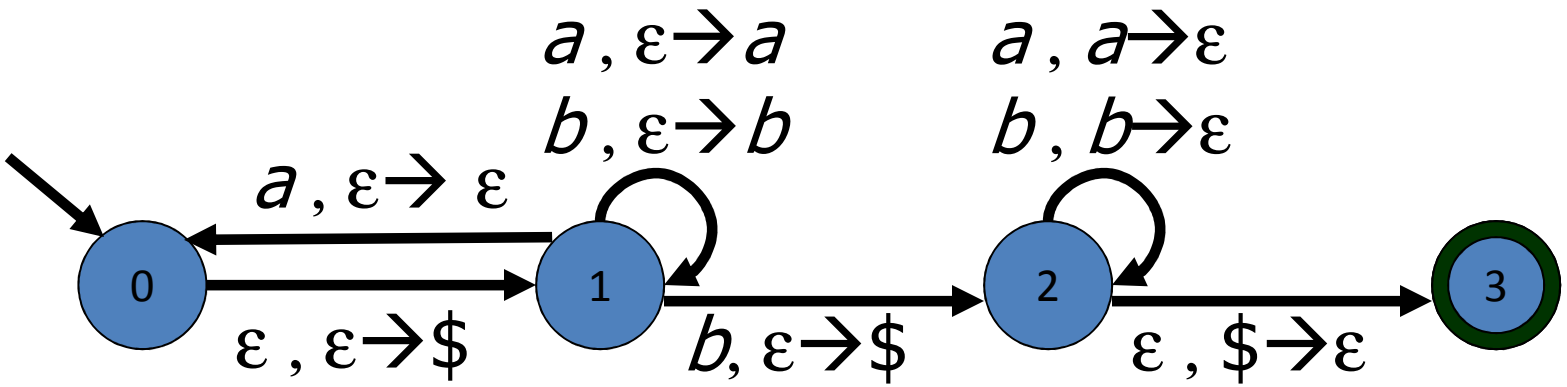
PDA
Definição Formal



Q: O que é $\delta(p,x,y)$ em cada caso?

1. $\delta(0,a,b)$
2. $\delta(0,\varepsilon,\varepsilon)$
3. $\delta(1,a,\varepsilon)$
4. $\delta(3,\varepsilon,\varepsilon)$

PDA
Definição Formal



R:

1. $\delta(0,a,b) = \varnothing$
2. $\delta(0,\varepsilon,\varepsilon) = \{(1,\$)\}$
3. $\delta(1,a,\varepsilon) = \{(0,\varepsilon),(1,a)\}$
4. $\delta(3,\varepsilon,\varepsilon) = \varnothing$

PDA - Exercício

- Forneça PDA´s para reconhecer as seguintes linguagens:
  - $\{a^n b^n \mid n \in \aleph\}$
  - $\{a^n b^{2n} \mid n \in \aleph\}$
  - $\{a^{2n} b^n \mid n \in \aleph\}$
  - $\{a^i b^j c^k \mid i = j \text{ ou } i=k\}$
  - $\{w\, w^R \mid w \in \{0,1\}^*\}$
  - $\{w \in \{0,1\}^* \mid$ o número de 0´s é igual ao de 1´s $\}$